SECTION-A

1

SETS

INTRODUCTION

George Cantor, a mathematician, born in Russia and educated in Germany, was the first to realise the importance of sets. The concept of a set is useful in almost every branch of mathematics. In this chapter, you will learn—

- the concept of a set
- representations of a set
- various types of sets
- set relations
- subsets of real numbers—intervals
- Venn diagrams
- operations on sets
- some basic results on cardinality of sets
- practical use of sets in solving problems.

1.1 SETS

In everyday life, we have to deal with *collections* or *aggregates* of objects of one kind or the other. For example, consider the following collections :

(*i*) the collection of even natural numbers less than 15 *i.e.* of the numbers 2, 4, 6, 8, 10, 12, 14.

(*ii*) the collection of vowels in the English alphabet *i.e.* of the letters *a*, *e*, *i*, *o*, *u*.

(*iii*) all colours of rainbow.

(*iv*) all states of India.

(*v*) all rivers of India.

(*vi*) all prime factors of 330 *i.e.* 2, 3, 5 and 11.

(*vii*) the roots of the equation $x^2 - 2x - 3 = 0$ *i.e.* 3 and -1.

(*viii*) all straight lines (drawn in a particular plane) passing through a given point.

We note that each one of the above collections is a *well-defined* collection of objects. By '*well defined collection of objects*' we mean that given a collection and an object, it should be possible to decide (beyond doubt) whether the object belongs to the given collection or not.

Set. *Any well-defined collection of objects is called a* **set**. *The objects of the set are called its* **members** *or* **elements**.

Thus, each one of the above collections is a set.

The terms 'objects', 'members' or 'elements' of a set are synonymous and are undefined.

Now, consider the collection of all good books on mathematics.

It is not a well-defined collection, since a mathematics book considered good by one person may not be considered good by another. So, this collection is not a set.

Note that the following collections are not well defined :

(*i*) all intelligent students of class XI of your school.

(*ii*) all big cities of India.

(*iii*) all beautiful girls of India.

(*iv*) five most renowned scientists of the world.

So, none of the above collections is a set.

The sets are usually denoted by capital letters A, B, C etc., and the members of the set are denoted by lower-case letters a, b, c etc.

If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A'). If x and y both belong to A, we write $x, y \in A$.

1.1.1 Representations of a set

There are two ways to represent a given set.

1. Roster or tabular form. In this form, we list all the members of the set within braces (curly brackets) and separate these by commas.

For example:

(*i*) the set A of all even natural numbers less than 15 in the *roster form* is written as
$A = \{2, 4, 6, 8, 10, 12, 14\}$.
Note that $2 \in A$, $10 \in A$ while $5 \notin A$.

(*ii*) the set S of vowels in the English alphabet in the *tabular form* is written as
$S = \{a, e, i, o, u\}$.

(*iii*) the set M of months of a year having less than 31 days in the *roster form* is written as
M = {February, April, June, September, November}.

(*iv*) the set L of letters in the word 'JODHPUR' in the *tabular form* is written as
L = {J, O, D, H, P, U, R}.

REMARKS

1. The order of listing the elements in a set can be changed. Thus, the set {3, 7, 8, 12} may also be written as {7, 3, 8, 12} or {12, 7, 3, 8} etc.
2. If one or more elements of a set are repeated, the set remains the same.
Thus, the set $\{a, b, c, b, b, a\}$ is the same as $\{a, b, c\}$.
3. While listing the elements of a set, it is sufficient to list its members only once.
Thus, the set X of letters in the word 'MATHEMATICS' in the tabular form is written as
X = {M, A, T, H, E, I, C, S}.
4. The roster form enables us to see all the members of a set at a glance. However, if the number of elements in a set is very large, then we represent the set by writing a few elements which clearly indicate the structure of the elements of the set followed (or preceded) by three dots and then writing the last element (if it exists).
Thus, the set A of odd natural numbers between 50 and 500 in the tabular form can be written as
$A = \{51, 53, 55, \ldots, 499\}$.
The set P of even integers less than 10 in the roster form can be written as
$P = \{\ldots, -4, -2, 0, 2, 4, 6, 8\}$.

2. Set builder form or rule method. In this form, we write a variable (say x) representing any member of the set which is followed by a colon ':' and thereafter we write the property satisfied by each member of the set and then enclose the whole description within braces. If A is a set consisting of elements x having property p, we write $A = \{x : x \text{ has property } p\}$, which is read as 'the set of elements x such that x has the property p'.

The colon ':' stands for the words 'such that'. Sometimes, we use the symbol '|' in place of the colon ':'.

For example:

(*i*) the set A of all even natural numbers less than 15 in the *builder form* is written as
A = {x : x is an even natural number less than 15}.

(*ii*) the set S of vowels in the English alphabet in the *builder form* is written as
S = {x : x is a vowel in the English alphabet}.

(*iii*) the set S = {1, 4, 9, 16, 25, ...} in the builder form can be written as
S = {x : x is the square of a natural number}.

1.1.2 Kinds of sets

1. Empty set. *A set which does not contain any element is called the* ***empty set*** *or the* ***null set*** *or the* ***void set***. There is only one such set.

It is denoted by φ or { }.

For example,

(*i*) the collection of natural numbers less than 1.

(*ii*) {x : $2x + 11 = 3$ and x is a natural number}.

(*iii*) {x : $x^2 = 9$ and x is an even integer}

(*iv*) {x : x is an even prime number greater than 2}.

Each one of these is the empty set.

2. Singleton set. *A set that contains only one element is called a* ***singleton*** *(or* ***unit****)* ***set***.

For example:

(*i*) {0}.

(*ii*) {x : $3x - 1 = 8$}.

(*iii*) {x : x is the capital of India}.

Each one of these is a singleton set.

3. Finite set. *A set that contains a limited (definite) number of different elements is called a* ***finite set***.

For example:

(*i*) S = {*a*, *e*, *i*, *o*, *u*}.

(*ii*) A = {2, 4, 6, ..., 100}.

(*iii*) S = {x : x is the capital of India}.

(*iv*) M = {x : x is a month of a year}.

(*v*) P = {x : $x \in$ **N** and x is a prime factor of 210} *i.e.* {2, 3, 5, 7}.

Each one of these is a finite set.

NOTE

As the empty set has no elements, φ is a finite set.

4. Infinite set. *A set that contains an unlimited number of different elements is called an* ***infinite set***. In other words, a set which is not finite is called an ***infinite set***.

For example:

(*i*) the set of even natural numbers *i.e.* {2, 4, 6, ...}.

(*ii*) {x : $x \in$ **N** and x is prime} *i.e.* {2, 3, 5, 7, 11, 13, ...}.

(*iii*) the set of all points on a line segment.

(*iv*) the set of all straight lines (drawn in a particular plane) passing through a given point.

Each one of these is an infinite set.

NOTE

All infinite sets cannot be written in the roster form. For example, the set of real numbers cannot be written in this form because the elements of this set do not follow any pattern.

1.1.3 Cardinal number (or order) of a finite set

The number of different elements in a finite set A is called the ***cardinal number*** *(or* ***order****) of A, and it is denoted by* $n(A)$ or $O(A)$.

For example:

(*i*) let $A = \{a, e, i, o, u\}$, then $n(A) = 5$.

(*ii*) let A be the set of letters in the word SCHOOL

i.e. $A = \{S, C, H, O, L\}$, then $n(A) = 5$.

(*iii*) let $A = \{x : x$ is a prime factor of $60\}$ *i.e.* $A = \{2, 3, 5\}$, then $n(A) = 3$.

(*iv*) let $D = \{x : x$ is a digit in our number system$\}$

i.e. $D = \{0, 1, 2, ..., 9\}$, then $n(D) = 10$.

NOTE

The cardinal number of the empty set is zero and the cardinal number of a singleton set is one. *The cardinal number of an infinite set is never defined.*

1.1.4 Some standard sets of numbers

(*i*) **Natural numbers.** The set of natural (or counting) numbers is denoted by **N**. Thus $N = \{1, 2, 3, ...\}$.

(*ii*) **Whole numbers.** The set of whole numbers is denoted by **W**. Thus $W = \{0, 1, 2, 3, ...\}$.

(*iii*) **Integers.** The set of all integers is denoted by I or **Z**. Thus $I = \{..., -2, -1, 0, 1, 2, ...\}$.

(*iv*) **Rational numbers.** Any number which can be expressed in the form $\frac{p}{q}$ where $p, q \in I$ and $q \neq 0$ is called a rational number. Thus $\frac{2}{3}, \frac{-7}{5}, 6, \frac{6}{8}, -7$ etc. are rational numbers. The set of rational numbers is denoted by **Q**.

(*v*) **Real numbers.** All rational as well as irrational numbers are real numbers. Thus $-3, 0, 5, \frac{5}{3}, -\frac{7}{2}, \sqrt{2}, -2+\sqrt{3}, \sqrt[3]{2}$ etc. are all real numbers. The set of real numbers is denoted by R.

(*vi*) **Irrational numbers.** The set of irrational numbers is denoted by T. Thus, $T = \{x : x \in R$ and $x \notin Q\}$ *i.e.* **T** is the set of all real numbers that are not rational. So, $\sqrt{2}, \sqrt{3}, -\sqrt[3]{5}, \pi$ are members of **T**.

(*vii*) **Positive rational numbers.** The set of positive rational numbers is denoted by Q^+.

(*viii*) **Positive real numbers.** The set of positive real numbers is denoted by R^+.

ILLUSTRATIVE EXAMPLES

Example 1. *State whether the statement 'collection of competent school teachers in Delhi is a set' is true or false. Justify your answer.*

Solution. False, because the collection of competent school teachers in Delhi is not well-defined. A particular teacher considered competent by one person might be considered incompetent by another.

Example 2. *Write the following sets in the roster form :*

(*i*) $A = \{x \mid x \in N$ *and* $4 < x \leq 10\}$.

(*ii*) $H = \{x \mid x$ *is a letter in the word 'ARITHMETIC'*$\}$.

(*iii*) $B = \{x \mid x \in N$ *and* $5 < x^2 < 50\}$.

(*iv*) $A = \{x : x \in I \text{ and } x^2 < 20\}$.

(*v*) $S = \{x : x$ *is a solution of the equation* $x^2 - x - 6 = 0\}$.

(*vi*) $B = \left\{x : x = \frac{2n-1}{n+2}, n \in W \text{ and } n < 4\right\}$.

(*vii*) $A = \{x : x$ *is a two digit number such that the sum of its digits is* $9\}$.

Solution. (*i*) A = {5, 6, 7, 8, 9, 10}.

(*ii*) H = {A, R, I, T, H, M, E, C}.

(*iii*) We know that the squares of natural numbers 3, 4, 5, 6, 7 lie between 5 and 50, therefore, the set A in roster form is A = {3, 4, 5, 6, 7}.

(*iv*) We know that the squares of integers 0, ± 1, ± 2, ± 3, ± 4 are less than 20, therefore, the set A in roster form is A = {–4, –3, –2, –1, 0, 1, 2, 3, 4}.

(*v*) The given equation is $x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\Rightarrow \quad x - 3 = 0 \text{ or } x + 2 = 0 \quad \Rightarrow x = 3, -2.$

∴ The set S in the roster form is S = {3, –2}.

(*vi*) As $n \in W$ and $n < 4$, $n = 0, 1, 2, 3$.

Also $x = \frac{2n-1}{n+2}$, putting $n = 0, 1, 2, 3$, we get

$$x = -\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, 1.$$

Therefore, the set A in roster form is $A = \left\{-\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, 1\right\}$.

(*vii*) As x is a two digit number and the sum of whose digits is 9, such numbers are 18, 27, 36, 45, 54, 63, 72, 81, 90.

Therefore, the set A in the roster form is

A = {18, 27, 36, 45, 54, 63, 72, 81, 90}.

Example 3. *Write the following sets in the roster form :*

(*i*) $D = \{t \mid t^3 = t,\ t \in R\}$.

(*ii*) $E = \left\{x : \frac{x-2}{x+3} = 3, x \in R\right\}$

(*iii*) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$.

Solution. (*i*) Given $D = \{t \mid t^3 = t, t \in R\}$.

$$t^3 = t \Rightarrow t^3 - t = 0 \Rightarrow t(t-1)(t+1) = 0$$

$\Rightarrow \quad t = 0, 1, -1, t \in R.$

∴ The set D in the roster form is D = {0, 1, –1}.

(*ii*) Given $E = \left\{x : \frac{x-2}{x+3} = 3, x \in R\right\}$.

$$\frac{x-2}{x+3} = 3 \Rightarrow 3x + 9 = x - 2$$

$\Rightarrow \quad 2x = -11 \Rightarrow x = -\frac{11}{2}, x \in R.$

∴ The set E in the roster form is $E = \left\{-\frac{11}{2}\right\}$.

(iii) Given F = $\{x \mid x^4 - 5x^2 + 6 = 0, x \in \mathbf{R}\}$.

$x^4 - 5x^2 + 6 = 0 \Rightarrow (x^2 - 2)(x^2 - 3) = 0$

$\Rightarrow \quad x^2 = 2, x^2 = 3$

$\Rightarrow \quad x = \pm\sqrt{2}, \pm\sqrt{3}, x \in \mathbf{R}$.

$\therefore$ The set F in the roster form is F = $\{\sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}\}$.

Example 4. *Write the following sets in the builder form :*

(i) *the counting numbers which are multiples of 6 and less than 50.*

(ii) *the fractions whose numerator is 1 and whose denominator is a counting number less than 10.*

(iii) *the set of all positive integers whose cube is odd.*

(iv) $A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{9}{10}\right\}$.

(v) $B = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots\right\}$.

Solution. (i) $\{x : x$ is a multiple of 6 and $0 < x < 50\}$.

(ii) $\left\{\frac{1}{x} : x \text{ is a counting number and } x < 10\right\}$.

(iii) Here, we are to consider only positive integers. As the cube of an even positive integer is an even positive integer and the cube of an odd positive integer is an odd positive integer, therefore, the members of the required set are all positive odd integers. Hence, in builder form the required set can be written as $\{x : x$ is an odd positive integer$\}$

i.e. $\{x : x = 2k + 1$ and $k \in \mathbf{W}\}$.

(iv) Here, we observe that each member in the given set has numerator one less than the denominator. Also the numerator begins with 1 and ends with 9.

Hence, the set A in the builder form can be written as

$$A = \left\{x : x = \frac{n}{n+1}, n \in \mathbf{N} \text{ and } 1 \le x \le 9\right\}.$$

(v) $$B = \left\{x : x = \frac{1}{n^2}, n \in \mathbf{N}\right\}.$$

Example 5. *Match each of the set on the left described in roster form with the same set on the right described in set builder form :*

(i) $\{3, 4, 5, 6, 7\}$ (a) $\{x : x$ *is a solution of* $x^2 + x - 2 = 0\}$

(ii) $\{1, 3, 5, 7, 9\}$ (b) $\{x : x$ *is a letter in the word TEACHER*$\}$

(iii) $\{A, C, H, R, T, E\}$ (c) $\{x : x$ *is an odd natural number less than* $10\}$

(iv) $\{1, -2\}$ (d) $\{x : x \in \mathbf{N}$ *and* $2 < x \le 7\}$.

Solution. In (d), $x \in \mathbf{N}$ and $2 < x \le 7$, so the values of x are 3, 4, 5, 6, 7 and hence (i) matches (d).

In (c), x is an odd natural number and less than 10, so the values of x are 1, 3, 5, 7, 9 and hence (ii) matches (c).

In (b), there are 7 letters in the word TEACHER and the letter E is repeated, so (iii) matches (b).

In (a), $x^2 + x - 2 = 0 \Rightarrow (x - 1)(x + 2) = 0 \Rightarrow x = 1, -2$, so (iv) matches (a).

Example 6. *State which of the following statements are true and which are false. Justify your answer.*

(i) $31 \notin \{x : x$ *has exactly two positive factors*$\}$.

(ii) $77 \in \{x : x$ *has exactly four positive factors*$\}$.

(iii) $28 \in \{x :$ *the sum of all positive factors of* x *is* $2x\}$.

(iv) $128 \in \{y :$ *the sum of all positive factors of* y *is* $2y\}$.

Solution. (*i*) False; since 31 has exactly two positive factors, 1 and 31, 31 belongs to the set.

(*ii*) True; since 77 has exactly four positive factors, 1, 7, 11 and 77, 77 belongs to the set.

(*iii*) True; since the sum of positive factors of 28 = 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 × 28.

(*iv*) False; since the sum of all positive factors of 128

$= 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255 \neq 2 \times 128.$

Example 7. *State which of the following sets are finite or infinite. In case of finite sets, mention the cardinal number :*

(*i*) $A = \{x : x \in N \text{ and } x^2 = 9\}$

(*ii*) $B = \{x : x \in W \text{ and } 2x - 1 = 0\}$

(*iii*) $C = \{x : x \in N \text{ and } x^2 - 3x + 2 = 0\}$

(*iv*) $D = \{x : x \in N \text{ and } x \text{ is prime}\}$

(*v*) $E = \{x : x \in N \text{ and } x \text{ is odd}\}$

(*vi*) $F = \{x : x \text{ is a month of a year having less than 31 days}\}$

(*vii*) $G = \{x : x \in I \text{ and } x > -3\}$.

Solution. (*i*) Given $x^2 = 9 \Rightarrow x = 3, -3$ but $x \in N$,

$\therefore$ A = {3}, which is a finite set. $n(A) = 1$.

(*ii*) Given $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ but $x \in W$,

$\therefore$ B = ϕ, which is a finite set. $n(\phi) = 0$.

(*iii*) Given $x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$ but $x \in N$,

$\therefore$ C = {1, 2}, which is a finite set. $n(C) = 2$.

(*iv*) $D = \{x : x \in N \text{ and } x \text{ is prime}\} = \{2, 3, 5, 7, 11, 13, \ldots\}$.

Since prime numbers are infinite in number, D is an infinite set.

(*v*) $E = \{x : x \in N \text{ and } x \text{ is odd}\} = \{1, 3, 5, 7, 9, 11, \ldots\}$.

Since odd numbers are infinite in number, E is an infinite set.

(*vi*) F = {x : x is month of a year having less than 31 days}

= {February, April, June, September, November}, which is a finite set.

$n(F) = 5$.

(*vii*) $G = \{x : x \in I \text{ and } x > -3\} = \{-2, -1, 0, 1, 2, 3, \ldots\}$, which is an infinite set.

Example 8. *If S = {x : x is a positive multiple of 3 less than 100} and P = {x : x is a prime number less than 20}, then write n(S) + n(P).*

Solution. Given S = {x : x is a positive multiple of 3 less than 100} and

P = {x : x is a prime number less than 20}.

The sets S and P in the roster form are :

S = {3, 6, 9, 12, ..., 99} and

P = {2, 3, 5, 7, 11, 13, 17, 19}

$\Rightarrow n(S) = 33$ and $n(P) = 8$.

$\therefore n(S) + n(P) = 33 + 8 = 41.$

EXERCISE 1.1

Very short answer type questions (1 to 8) :

1. State which of the given collection of objects is a set :

(*i*) A collection of popular cinema actors of India.

(*ii*) The collection of even natural numbers less than 51.

(*iii*) The collection of counting numbers less than 1.

(*iv*) Collection of interesting books written by Shakespeare.

(v) The collection of novels written by Munshi Prem Chand.

(vi) The collection of 10 most talented students of your school.

(vii) Collection of all rivers flowing in India.

(viii) Collection of 5 rivers flowing in India.

(ix) Collection of all rational numbers which lie between -1 and 1.

(x) A team of eleven best cricketers of the world.

(xi) A collection of most dangerous animals of the world.

2. If A = {3, 5, 7, 9, 11}, then write which of the following statements are true. If a statement is not true, mention why.

(i) $3 \in A$ (ii) $5, 9 \in A$ (iii) $8 \notin A$

(iv) $9 \notin A$ (v) $\{3\} \in A$ (vi) $\{5, 7\} \in A$.

3. Use the roster method to represent the following sets :

(i) The counting numbers which are multiples of 6 and less than 50.

(ii) The fractions whose numerator is 1, and whose denominator is a counting number less than 10.

(iii) $\{x : x \in N$ and x is a prime factor of $84\}$.

(iv) The set of odd integers lying between -4 and 8.

(v) The set of all natural numbers x for which $x + 6$ is less than 10.

(vi) The set of all integers x for which $x + 6$ is greater than 10.

(vii) The set of all integers x for which $x + 6$ is less than 10.

(viii) The set of all integers x for which $\frac{60}{x}$ is a natural number.

(ix) $\left\{x : x \in I, -\frac{1}{2} < x < \frac{9}{2}\right\}$.

(x) $\{x : x \in N$ and $4x - 3 \le 15\}$.

(xi) $\{x : x \in N, x^2 < 40\}$

(xii) $\{x : x \in Z$ and $x^2 < 16\}$.

(xiii) The set of all digits in our *number system*.

(xiv) The set of all letters in the word TRIGONOMETRY.

(xv) The set of all vowels in the English alphabet which precede q.

(xvi) $\{x : x$ is a consonant in the English alphabet which precedes $k\}$.

4. Write the following sets in the builder form :

(i) {1, 3, 5, 7, 9, 11, 13} (ii) {2, 4, 6, 8, ...}

(iii) {3, 6, 9, 12, 15} (iv) {2, 4, 8, 16, 32, 64}

(v) {5, 25, 125, 625} (vi) {1, 4, 9, 16, ..., 100}

(vii) {1, 4, 9, 16, 25, ...} (viii) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$.

5. Which of the following are examples of the null set?

(i) Set of even prime numbers.

(ii) Set of odd natural numbers divisible by 2.

(iii) Set of all Indian kids 5 metres tall.

(iv) $\{x : x \in N, x < 5$ and $x > 8\}$.

(v) $\{x : x$ is a point common to any two parallel straight lines$\}$.

(vi) $\{x : x$ is a student of your school presently studying in both classes XI and XII$\}$.

6. Which of the following sets are finite or infinite?

(i) The set of days of a week.

(ii) The set of numbers which are multiples of 7.

(iii) The set of animals living on Earth.

(*iv*) The set of consonants in the English alphabet.

(*v*) The set of circles drawn in a plane.

(*vi*) The set of prime numbers which are less than one crore.

7. Find the cardinal number of the following sets :

(*i*) { } (*ii*) {0} (*iii*) A = {1, 2, 2, 1, 3}

(*iv*) The set of all Indians having 8 legs.

(*v*) The set of all letters in the word PRINCIPAL

(*vi*) The set of all vowels in the word PRINCIPAL.

8. (*i*) Write the cardinal number of the set A, where A = {x : x is a two digit number, sum of whose digits is 8}.

(*ii*) Write the cardinal number of the set of all integers x for which $\frac{30}{x}$ is a natural number.

(*iii*) What is the cardinal number of the set X, where X = {x : x is a letter in the word 'CHANDIGARH'}?

9. Match each of the sets on the left described in roster form with the same set on the right described in set builder form :

(*i*) {2, 3} (*a*) {x : $x \in$ N and is a divisor of 6}

(*ii*) {5, –5} (*b*) {x : $x \in$ N and is a prime divisor of 6}

(*iii*) {1, 3, 5} (*c*) {x : x is a letter in the word LITTLE}

(*iv*) {1, 2, 3, 6} (*d*) {x : x is an odd natural number less than 6}

(*v*) {T, E, L, I} (*e*) {x : x is a root of the equation $x^2 - 25 = 0$}.

10. State which of the following statements are true and which are false. Justify your answer.

(*i*) $37 \notin$ {x : x has exactly two positive factors}

(*ii*) $35 \in$ {x : x has exactly four positive factors}

(*iii*) $496 \in$ {y : the sum of all positive factors of y is $2y$}.

(*iv*) $3 \notin \{x : x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

Hint. (*iii*) $496 = 2^4 \times 31$, so all the positive factors of 496 are

1, 2, 4, 8, 16, 31, 62, 124, 248 and 496.

The sum of all the positive factors of 496

$= 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496 = 992 = 2 \times 496$

$\Rightarrow$ $496 \in$ {y : the sum of all positive factors of y is $2y$}.

Hence, the given statement is true.

11. Classify the following sets into finite set and infinite set. In case of finite sets, mention the cardinal number.

(*i*) A = {x : $x \in$ I, $x < 5$}.

(*ii*) A = {x : $x \in$ W, x is divisible by 4 and 9}.

(*iii*) P = {x : x is an even prime number > 2}.

(*iv*) F = {x : $x \in$ N and x is a factor of 84}.

(*v*) B = {x : x is a two digit number, sum of whose digits is 12}.

(*vi*) C = {x : $x \in$ W, $3x - 7 \leq 8$}.

(*vii*) {x : $x = 5n$, $n \in$ N and $x < 20$}.

(*viii*) {x : $x = 5n$, $n \in$ I and $x < 20$}.

(*ix*) $\left\{x : x = \frac{n}{n+1}, n \in W \text{ and } n \leq 10\right\}$.

(*x*) $\left\{x : x = \frac{2n}{n+3}, n \in N \text{ and } 5 < n < 20\right\}$.

1.2 SET RELATIONS

1.2.1 Equivalent sets

Two (finite) sets A and B are called equivalent if they have the same number of elements.

Thus two finite sets A and B are equivalent, written as $A \leftrightarrow B$ (read as A is equivalent to B), if $n(A) = n(B)$.

For example :

(i) Let $A = \{a, b, c, d, e\}$ and $B = \{2, 3, 5, 7, 9\}$, then $n(A) = 5 = n(B)$. So $A \leftrightarrow B$.

(ii) Let $A = \{x : x$ is a colour of rainbow$\}$ and $B = \{x \mid x \in W, x < 7\}$, then $n(A) = 7 = n(B)$. So $A \leftrightarrow B$.

(iii) Let $P = \{x : x$ is a letter in the word 'FLOWER'$\}$ and $Q = \{x : x$ is a letter in the word 'FOLLOWER'$\}$, then $n(P) = 6 = n(Q)$ because each set $= \{F, L, O, W, E, R\}$. So $P \leftrightarrow Q$.

1.2.2 Equal sets

Two sets A and B are said to be equal if they have exactly the same elements. We write it as $A = B$.

Thus $A = B$ if every member of A is a member of B and every member of B is a member of A. If A and B are not equal, we write it as $A \neq B$.

For example :

(i) $A = \{1, 2\}$ and $B = \{2, 1, 1, 2, 1\}$, then $A = B$.

(ii) Let $P = \{x ; x$ is a vowel in the word 'EQUALITY'$\}$ and $Q = \{x : x$ is a vowel in the word 'QUATITATIVE'$\}$, then $P = Q$ because each set $= \{E, U, A, I\}$.

(iii) Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $B = \{x : x \in I$ and $x^2 < 10\}$, then $A = B$.

(iv) Let $P = \{x : x \in N$ and $x^2 - 2 = 0\}$ and $Q = \{x : x$ is a triangle having 5 sides$\}$, then $P = Q$ because each set $= \phi$.

REMARK

If A, B are finite sets and $A = B$, then $n(A) = n(B)$ so $A \leftrightarrow B$ *i.e.* two finite equal sets are always equivalent but the converse may not be true. For example, let $A = \{2, 3, 5\}$ and $B = \{2, 3, 4\}$ then $n(A) = 3 = n(B)$, so $A \leftrightarrow B$ but $A \neq B$.

Thus, two (finite) equal sets are always equivalent but two equivalent sets may not be equal.

1.2.3 Subset

Let A, B be any two sets, then A is called a subset of B if every member of A is also a member of B. We write it as $A \subset B$ (read as 'A is a subset of B' or 'A is contained in B').

Thus $A \subset B$ *if* $x \in A$ *implies* $x \in B$.

If $A \subset B$ *i.e.* A is contained in B, we may also say that B contains A or B is a **superset** of A. We write it as $B \supset A$ (read as 'B contains A' or 'B is a superset of A').

If there exists atleast one element in A which is not a member of B, then A is not a subset of B and we write it as $A \not\subset B$.

For example :

(i) let $A = \{2, 3, 5\}$ and $B = \{1, 2, 3, 5, 6\}$. Since every member of A is also a member of B, $A \subset B$. Note that $1 \in B$ but $1 \notin A$, so $B \not\subset A$.

(ii) let $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d, e\}$. Note that $i \in A$ but $i \notin B$, so $A \not\subset B$. Also $b \in B$ but $b \notin A$, so $B \not\subset A$.

(iii) let P be the set of letters in the word 'SCHOOL' and Q be the set of letters in the word 'SCHOLAR'. In roster form, $P = \{S, C, H, O, L\}$ and $Q = \{S, C, H, O, L, A, R\}$. Clearly $P \subset Q$ while $Q \not\subset P$.

(iv) let A = {x : x is a divisor of 56} and B = {x : x is a prime divisor of 56}, then

A = {1, 2, 4, 7, 8, 14, 28, 56} and B = {2, 7}. It is easy to see that B ⊂ A while A ⊄ B.

(v) let A = {1, 3, 5, 3, 1} and B = {x : x is an odd natural number less than 6}.

It is easy to see that A ⊂ B and B ⊂ A. In fact, A = B.

Proper subset. *Let A be any set and B be a non-empty set, then A is called a proper subset of B if every member of A is also a member of B and there exists atleast one element in B which is not a member of A.*

If A is a proper subset of B, we write it as A ⊂ B, A ≠ B.

In the above example, in (i) A is a proper subset of B, in (iii) P is a proper subset of Q and in (iv) B is a proper subset of A.

REMARK

If two sets A and B are equal *i.e.* A = B, then A ⊂ B and B ⊂ A. Conversely, if A ⊂ B and B ⊂ A, then A = B. Thus A = B if and only if for every $a \in A \Rightarrow a \in B$ and for every $b \in B \Rightarrow b \in A$.

NOTE

Let A be any set, then

(1) A ⊂ A *i.e.* every set is a subset of itself, but not a proper subset. A subset which is not a proper subset is called an improper **subset.**

(2) Every set has only one *improper subset*.

(3) Since the empty set has no elements, φ ⊂ A *i.e.* the empty set is a subset of every set.

(4) Empty set is a proper subset of every set except itself.

Subsets of a set

(i) Let A = {a}, then the subsets of A are φ, A.

Note that n (A) = 1, number of subsets of A = 2 = 2^1.

(ii) Let A = {1, 2}, then the subsets of A are φ, {1}, {2}, A. Note that n (A) = 2, number of subsets of A = 4 = 2^2.

(iii) Let A = {1, 2, 3}, then the subsets of A are

φ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, A.

Note that n (A) = 3, number of subsets of A = 8 = 2^3.

REMARK

If A is a set with n (A) = m, then the number of subsets of A = 2^m and the number of proper subsets of A = $2^m - 1$.

1.2.4 Power set

The set formed by all the subsets of a given set A is called the power set of A, it is denoted by P(A).

For example, let A = {1, 2}, then P(A) = {φ, {1}, {2}, A}.

REMARK

If A is a set with n (A) = m, then n (P(A)) = 2^m.

1.2.5 Universal set

A set that contains all the elements under consideration in a given problem is called universal set. It is denoted by ξ or U.

It is a kind of 'parent set'. Every set under discussion is a subset of universal set.

Note that the choice of universal set is not unique. Universal set may vary from one problem to another. Therefore, we shall always specify universal set in a given problem.

For example :

(*i*) For A = {*b*, *c*, *g*, *m*, *n*}, universal set may be
{*x* : *x* is a letter in English alphabet}.

(*ii*) For A = {$x : x \in$ N, $3 \le x < 12$}, universal set may be
{1, 2, 3, ..., 20} or **N**.

(*iii*) For A = {Earth, Mars}, universal set may be
{*x* : *x* is a planet of our solar system}.

1.2.6 Subsets of real numbers

We know some standard sets of numbers. These sets are subsets of the set of real numbers. It is easy to see that :

$$N \subset W \subset I \subset Q \subset \mathbf{R},\ T \subset \mathbf{R},\ N \not\subset T.$$

Also $Q' \subset Q \subset \mathbf{R}$ and $\mathbf{R}^+ \subset \mathbf{R}$.

Intervals as subsets of R

Intervals are some special types of subsets of the set of real numbers.

Let *a* and *b* be two (distinct) real numbers and $a < b$.

The set of all real numbers lying between a and b is said to form an ***open interval***. It is denoted by (a, b). Precisely,

$$(a, b) = \{x : x \in \mathbf{R},\ a < x < b\}.$$

The number *a* is called the **left end point** of the interval and the number *b* is called the **right end point**. It may be noted that the open interval (a, b) does not contain the left and right end points *a* and *b*.

Geometrically, let the points A and B on the real axis represent the real numbers *a* and *b* respectively, then the open interval (a, b) is the set of all points to the right of A and to the left of B. It is represented on the real axis as follows :

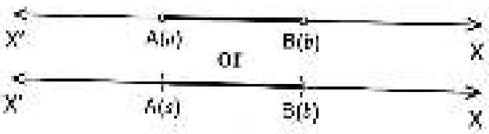


Fig. 1.1.

The set of all real numbers lying between a and b and including the numbers a and b is said to form a ***closed interval***. It is denoted by $[a, b]$. Precisely,

$$[a, b] = \{x : x \in \mathbf{R},\ a \le x \le b\}.$$

The closed interval $[a, b]$ is represented on the real axis as follows :

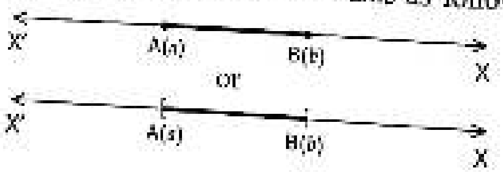


Fig. 1.2.

The set of all real numbers lying between a and b, and including the number b is said to form an ***open-closed interval***. This interval is open on the left but closed on the right, it is denoted by $(a, b]$. Precisely,

$$(a, b] = \{x : x \in \mathbf{R},\ a < x \le b\}.$$

It is represented on the real axis as follows :

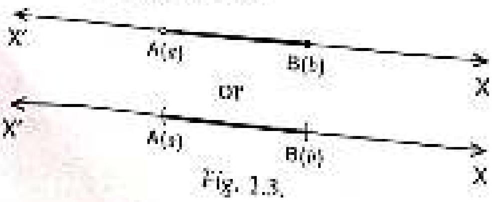


Fig. 1.3.

The set of all real numbers lying between a and b, and including the number a is said to form a closed-open interval. This interval is closed on the left but open on the right, it is denoted by $[a, b)$. Precisely,

$$[a, b) = \{x : x \in \mathbf{R}, a \le x < b\}.$$

It is represented on the real axis as follows :

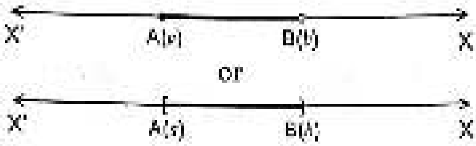


Fig. 1.4.

The number $(b - a)$ is called the **length** of any of the intervals (a, b), $[a, b]$, $(a, b]$ or $[a, b)$.

The intervals introduced above are all **finite intervals**. We shall also need **infinite** intervals.

*The set of all real numbers x such that x > a form an **infinite interval**.* It is denoted by (a, ∞). Precisely,

$$(a, \infty) = \{x : x \in \mathbf{R}, x > a\}.$$

It is represented on the real axis as follows :

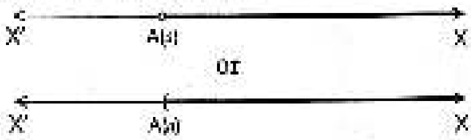


Fig. 1.5.

*The set of all real numbers x such that x ≥ a form an **infinite interval**.* It is denoted by $[a, \infty)$. Precisely,

$$[a, \infty) = \{x : x \in \mathbf{R}, x \ge a\}.$$

It is represented on the real axis as follows :

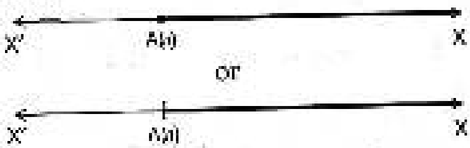


Fig. 1.6.

*The set of all real numbers x such that x < a form an **infinite interval**.* It is denoted by $(-\infty, a)$. Precisely,

$$(-\infty, a) = \{x : x \in \mathbf{R}, x < a\}.$$

It is represented on the real axis as follows :

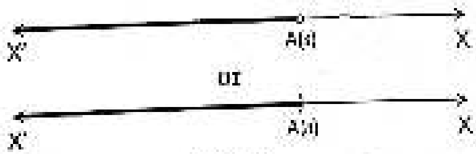


Fig. 1.7.

*The set of all real numbers x such that x ≤ a form an **infinite interval**.* It is denoted by $(-\infty, a]$. Precisely,

$$(-\infty, a] = \{x : x \in \mathbf{R}, x \le a\}.$$

It is represented on the real axis as follows :

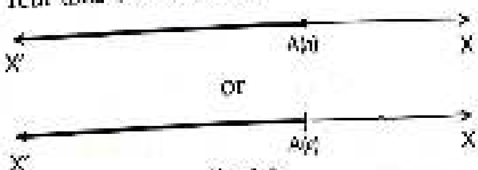


Fig. 1.8.

*The set of all real numbers is an **infinite interval**.* It is denoted by $(-\infty, \infty)$. Precisely,

$$(-\infty, \infty) = \{x : x \in \mathbf{R}\}.$$

The real axis itself represents this interval.

Fig. 1.9.

REMARK

It may be noted that '∞' (read as infinity) *is not a number* and cannot be treated as such. It is a symbol representing largeness without any bound *i.e.* greater than any positive real number however large.

Similarly, '−∞' represents smallness without any bound *i.e.* smaller than any negative real number however small.

ILLUSTRATIVE EXAMPLES

Example 1. *Find all pairs of equal sets (if any) :*

$A = \{0\}$, $B = \{x : x < 5 \text{ and } x > 15\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x : x$ *is a positive integral root of the equation* $x^2 - 2x - 15 = 0\}$.

Solution. The given sets are :

A = {0}, B = φ

C = {5}, D = {5, −5}

E = {5} $(\because x^2 - 2x - 15 = 0 \Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x = 5, -3)$

Here, we find that the only pair of equal sets is C and E.

Example 2. *Consider the following sets :*

φ, $A = \{1, 3\}$, $B = \{1, 5, 9\}$ *and* $C = \{1, 2, 3, 5, 7, 9\}$.

Insert the correct symbol ⊂ *or* ⊄ *between the following pairs of sets :*

(*i*) φ ... B (*ii*) A ... B (*iii*) A ... C (*iv*) B ... C.

Solution. (*i*) Since φ is subset of every set, φ ⊂ B.

(*ii*) Since 3 ∈ A and 3 ∉ B, therefore, A ⊄ B.

(*iii*) Since every member of A is a member of C, A ⊂ C.

(*iv*) Since every member of B is member of C, B ⊂ C.

Example 3. *State whether each of the following statements is true or false for the sets A, B and C where*

$A = \{x : x$ *is a letter of the word* 'BOWL'$\}$

$B = \{x : x$ *is a letter of the word* 'ELBOW'$\}$

$C = \{x : x$ *is a letter of the word* 'BELLOW'$\}$

(*i*) $A \subset B$ (*ii*) $B \supset C$ (*iii*) $B = C$ (*iv*) $B \leftrightarrow C$

(*v*) *A is a proper subset of B* (*vi*) *B is a proper subset of C.*

Solution. The given sets in the roster form are

A = {B, O, W, L},

B = {E, L, B, O, W} and

C = {B, E, L, O, W}

(*i*) true (*ii*) true (*iii*) true (*iv*) true (*v*) true (*vi*) false.

Example 4. *Let* $\xi = \{1, 2, 3, \ldots, 50\}$, $A = \{x : x$ *is divisible by 2 and 3*$\}$,

$B = \{x : x = n^2, n \in N\}$ *and* $C = \{x : x$ *is a factor of 42*$\}$, *then*

(*i*) *write the sets A, B and C in roster form.*

(*ii*) *state* $n(A)$, $n(B)$ *and* $n(C)$.

(*iii*) *state whether* $A \leftrightarrow B$.

(*iv*) *state whether* $A \leftrightarrow C$.

Solution. (*i*) Here $\xi = \{1, 2, 3, ..., 50\}$.

It is understood that A, B and C are subsets of ξ, so the members of these sets are to be taken only from ξ.

The sets A, B and C in roster form are

A = {6, 12, 18, 24, 30, 36, 42, 48},

B = {1, 4, 9, 16, 25, 36, 49} and

C = {1, 2, 3, 6, 7, 14, 21, 42}.

(*ii*) Note that $n(A) = 8$, $n(B) = 7$ and $n(C) = 8$.

(*iii*) No; because $n(A) \neq n(B)$.

(*iv*) Yes; because $n(A) = n(C)$. Note that $A \neq C$.

Example 5. *Let A = {x : x is a letter in the word 'GEORGE CANTOR'} and B = {x : x is a vowel in the word 'GEORGE CANTOR'}, then*

(*i*) *write the sets A, B in the tabular form.* (*ii*) *state n (A) and n (B).*

(*iii*) *write the number of proper subsets of A.* (*iv*) *write the power set of B.*

Solution. (*i*) A = {G, E, O, R, C, A, N, T} and B = {E, O, A}.

(*ii*) n (A) = 8 and n (B) = 3.

(*iii*) The number of proper subsets of A = $2^8 - 1 = 256 - 1 = 255$.

(*iv*) P(B) = {ϕ, {E}, {O}, {A}, {E, O}, {E, A}, {O, A}, B}.

Example 6. *Let A = {1, 2, 3}, B = {2, 4} and C = {1, 2, 3, 4}. Find all sets X such that*

(*i*) $X \subset A$ *and* $X \subset B$

(*ii*) $X \subset C$ *but* $X \not\subset A$.

Solution. (*i*) Subsets of A are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3} and {1, 2, 3}; subsets of B are ϕ, {2}, {4} and {2, 4}.

$X \subset A$ and $X \subset B$ means that X is a subset of both A and B. Hence X = ϕ, {2}.

(*ii*) Subsets of C are ϕ, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4} and {1, 2, 3, 4}.

$X \subset C$ but $X \not\subset A$ means that X is a subset of C but X is not a subset of A.

Hence X = {4}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}.

Example 7. *Write down the power set of A where :*

(*i*) $A = \phi$ (*ii*) $A = \{0\}$ (*iii*) $A = \{0, \{0, 1\}\}$.

Also find the cardinal number of P(A) in each case.

Solution. (*i*) Here, A = ϕ.

We know that the only subset of ϕ is ϕ, therefore,

P(A) = {ϕ}.

∴ Cardinal number of P(A) = 1.

(*ii*) Here, A = {0}.

The subsets of A are ϕ and A, therefore,

P(A) = {ϕ, {0}}.

∴ Cardinal number of P(A) = 2.

(*iii*) Here A = {0, {0, 1}} = {0, B} where B = {0, 1}.

The subsets of A are ϕ, {0}, {B} and A, therefore,

P(A) = {ϕ, {0}, {{0, 1}}, {0, {0, 1}}}.

∴ Cardinal number of P(A) = 4.

Example 8. *For any sets A and B, is it true that* $P(A) \cup P(B) = P(A \cup B)$? *Justify your answer.*

Solution. Let $A = \{1, 2\}$ and $B = \{2, 3\}$, then

$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$ and $P(B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$,

$\therefore$ $P(A) \cup P(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{3\}, \{2, 3\}\}$.

Now $A \cup B = \{1, 2, 3\}$,

$\therefore$ $P(A \cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

Note that $P(A) \cup P(B) \neq P(A \cup B)$.

Example 9. *Two finite sets A and B have m and k elements respectively. If the ratio of cardinal number of power set of A to cardinal number of power set of B is 64 : 1 and* $n(A) + n(B) = 12$, *then find the values of m and k.*

Solution. Given $n(A) = m$ and $n(B) = k$

$\Rightarrow$ $n(P(A)) = 2^m$ and $n(P(B)) = 2^k$.

As ratio of $n(P(A))$ to $n(P(B)) = 64 : 1$,

$\therefore$ $\frac{2^m}{2^k} = \frac{64}{1} \Rightarrow 2^{m-k} = 64$

$\Rightarrow$ $2^{m-k} = 2^6 \Rightarrow m - k = 6$ ...(i)

Also $n(A) + n(B) = 12 \Rightarrow m + k = 12$ ...(ii)

Solving (*i*) and (*ii*) simultaneously, we get $m = 9$ and $k = 3$.

Hence, $m = 9$ and $k = 3$.

Example 10. *Two finite sets have m and k elements. If the total number of subsets of first set is 56 more than the total number of subsets of second set, then find the values of m and k.*

Solution. Let the two sets be A and B, then $n(A) = m$ and $n(B) = k$, $m > k$.

Now $n(P(A)) = 2^m$ and $n(P(B)) = 2^k$.

According to given, $2^m - 2^k = 56$

$\Rightarrow$ $2^k (2^{m-k} - 1) = 2^3 \times 7$

$\Rightarrow$ $2^k = 2^3$ and $2^{m-k} - 1 = 7$

$\Rightarrow$ $k = 3$ and $2^{m-3} = 8$

$\Rightarrow$ $k = 3$ and $2^{m-3} = 2^3 \Rightarrow k = 3$ and $m - 3 = 3$

$\Rightarrow$ $k = 3$ and $m = 6$.

Hence, $m = 6$ and $k = 3$.

Example 11. *If A, B and C are sets, prove that*

(*i*) $A \subset B, B \subset C \Rightarrow A \subset C$

(*ii*) $A \subset B, B \subset C, C \subset A \Rightarrow A = C$

(*iii*) $A \subset \phi \Leftrightarrow A = \phi$.

Solution. (*i*) Given $A \subset B$ and $B \subset C$. To prove $A \subset C$, consider any arbitrary element $x \in A$. Then

$x \in A \Rightarrow x \in B$ ($\because A \subset B$)

$\Rightarrow$ $x \in C$ ($\because B \subset C$)

Hence, $A \subset C$.

(*ii*) $A \subset B, B \subset C \Rightarrow A \subset C$ (see part (*i*))

Also $C \subset A$ (given)

Thus, $A \subset C$ and $C \subset A \Rightarrow A = C$.

(*iii*) Let $A \subset \phi$, we shall prove that $A = \phi$.

We know that empty set is always a subset of every set *i.e.* $\phi \subset A$. Thus, we get $\phi \subset A$ and $A \subset \phi$ (given)

$\Rightarrow$ $A = \phi$.

Conversely, let $A = \phi \Rightarrow A \subset \phi$ and $\phi \subset A$.

In particular, $A \subset \phi$.

Therefore, $A \subset \phi \Leftrightarrow A = \phi$.

EXERCISE 1.2

Very short answer type questions (1 to 14) :

1. In the following, state whether A = B or not :
   (*i*) $A = \{x : x + 2 = 3\}$, $B = \{x : x \in N$ and is less than $2\}$
   (*ii*) $A = \{x : x \in N$ and $3x - 1 < 2\}$, $B = \{x : x \in W$ and $3x - 1 < 2\}$
   (*iii*) $A = \{x : x \in N$ and is a prime factor of $36\}$, $B = \{1, 2, 3, 4, 6, 9, 12\}$
   (*iv*) $A = \{x : x \in I$ and $x^2 \leq 4\}$, $B = \{x : x \in R$ and $x^2 - 3x + 2 = 0\}$
   (*v*) $A = \{2, 3\}$ and $B = \{x : x$ is a solution of $x^2 + 5x + 6 = 0\}$
   (*vi*) $A = \{x : x$ is a letter in the word LOYAL$\}$ and
   $B = \{y : y$ is a letter in the word ALLOY$\}$
   (*vii*) $A = \{x : x$ is a letter in the word WOLF$\}$ and
   $B = \{y : y$ is a letter in the world FOLLOW$\}$
   (*viii*) $A = \{10, 20, 30, 40, 50, ...\}$ and $B = \{x : x$ is a multiple of $10\}$.
2. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3, 4\}$ and $C = \{2, 4, 5\}$; state whether the following statements are true or false.
   (*i*) $A \subset B$ (*ii*) $A \subset C$ (*iii*) $B \subset A$ (*iv*) $B \subset C$
   (*v*) $C \subset A$ (*vi*) $C \subset B$ (*vii*) $B \sim C$ (*viii*) $\phi \subset B$
   (*ix*) $A \leftrightarrow B$ (*x*) $B \leftrightarrow C$ (*xi*) $A \leftrightarrow C$.
3. Use appropriate symbol '$\in, \notin, \subset, \supset, =$' to fill in the blanks :
   (*i*) 4 ...... $\{1, 2, 3, 4\}$ (*ii*) -5 ...... $\{2, 3, 4, 5, 6\}$
   (*iii*) $\{2\}$ ...... $\{2, 3, 4\}$ (*iv*) $\{a, b, c\}$ ...... $\{b, a, c\}$
   (*v*) $\{a, b, c\}$ ...... $\{a, b, b, a, c\}$ (*vi*) $\{a, i, u\}$ ...... $\{a, a, i\}$
   (*vii*) MUMBAI ... $\{x : x$ is a capital city of countries in Asia$\}$.
4. Examine whether the following statements are true or false :
   (*i*) $\{a, b\} \not\subset \{b, c, a\}$ (*ii*) $\{1, 2, 3\} \subset \{1, 3, 5\}$
   (*iii*) $\{a\} \subset \{a, b, c\}$ (*iv*) $\{a\} \in \{a, b, c\}$
   (*v*) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$
   (*vi*) $\{x : x$ is an even natural number less than $8\} \subset \{x : x$ is a natural number which divides $36\}$.
5. State whether each of the following statements is true or false :
   (*i*) $2 \subset \{1, 2, 3\}$ (*ii*) $\{2\} \in \{1, 2, 3\}$ (*iii*) $\phi \subset \{1, 2, 3\}$
   (*iv*) $0 \in \phi$ (*v*) $\{1, 2\} \subset \{1, \{2, 3\}\}$ (*vi*) $\phi \subset \{\phi\}$.
6. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are not true and why?
   (*i*) $\{3, 4\} \subset A$ (*ii*) $\{3, 4\} \in A$ (*iii*) $\{\{3, 4\}\} \subset A$
   (*iv*) $\{1, 2, 5\} \subset A$ (*v*) $\{1, 2, 5\} \in A$ (*vi*) $\{1, 2, 3\} \subset A$
   (*vii*) $1 \in A$ (*viii*) $1 \subset A$ (*ix*) $\phi \in A$
   (*x*) $\phi \subset A$ (*xi*) $\{\phi\} \subset A$.
7. If $\xi = \{1, 2, 3, ..., 10\}$ and $A = \{x \mid x$ is a prime factor of $66\}$. List the set A.
8. Write down all the subsets of the following sets :
   (*i*) $\{a\}$ (*ii*) $\{a, b\}$ (*iii*) ϕ.
9. Write the power set of A, where $A = \{-1, 0, 2\}$.
10. Write the number of proper subsets of A, where $A = \{-3, -1, 0, 1, 4\}$.

11. If $\xi = \{1, 2, 3, ..., 40\}$ and $A = \{x : x$ is a factor of $42\}$, then write $n(A)$.
12. If ξ = {all digits in our number system} and $A = \{x : x$ is a multiple of $3\}$, then write $n(A)$.
13. Write the following sets as intervals :
    (*i*) $\{x : x \in R, -3 < x \le 5\}$ (*ii*) $\{x : x \in R, -5 < x < -1\}$
    (*iii*) $\{x : x \in R, 2 \le x \le 7\}$ (*iv*) $\{x : x \in R, 0 \le x < 3\}$
    (*v*) $\{x : x \in R, x \le 5\}$ (*vi*) $\{x : x \in R, x < -3\}$
    (*vii*) $\{x : x \in R, x \ge -2\}$.
14. Write the following intervals in set builder form :
    (*i*) $(-2, 0]$ (*ii*) $(2, 7)$ (*iii*) $[-5, -2]$ (*iv*) $[-9, 4)$
    (*v*) $(3, \infty)$ (*vi*) $(-\infty, -1]$ (*vii*) $(-\infty, 4)$.
15. If $\xi = \{1, 2, 3, ..., 12\}$, $A = \{x \mid 2x + 3 \le 18\}$ and $B = \{x \mid x^2 \le 40\}$; write A and B in the roster form.
16. Let $\xi = \{0, 1, 2, 3, ..., 50\}$, $A = \{x : x = 6n, n \in W\}$
    $B = \{x : x = 7n, n \in W\}$ and $C = \{x : x$ is a factor of $72\}$, then
    (*i*) write the sets A, B and C in roster form
    (*ii*) state $n(A)$, $n(B)$ and $n(C)$.
17. Given sets $A = \{1, 3, 5\}$, $B = \{0, 2, 4, 6, 8\}$ and $C = \{2, 3, 4, 6\}$, which of the following may be taken as universal set(s) for all the three sets A, B and C?
    (*i*) $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (*ii*) $\{0, 1, 2, 3, 4, 5, 6\}$
    (*iii*) ϕ (*iv*) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
18. Given $A = \{x : x$ is a letter in the word ACCUMULATOR$\}$
    (*i*) Express A in roster form.
    (*ii*) Find the cardinal number of the set of vowels in A.
    (*iii*) Write down the power set of the set of vowels in A.
19. List all the proper subsets of $\{0, 1, 2, 3\}$.
20. If the set A has five elements, how many subsets will A have ? If A has six elements, how many proper subsets will A have?
21. Two finite sets have m and k elements. If the number of subsets of the first set is 112 more than the number of subsets of the second set, then find the values of m and k.

1.3 VENN DIAGRAMS

Most of the ideas about sets and the various relationships between them can be visualised by means of geometric figures known as '**Venn diagrams**' (or **Venn-Euler diagrams**). Usually, the universal set ξ is denoted by a rectangle and its subsets by closed curves within the rectangle, such as circles, ovals (ellipses) etc.

If A, B are the sets whose members are represented by points within circles, then the Venn diagram (fig. 1.10) represents that $A \subset B$.

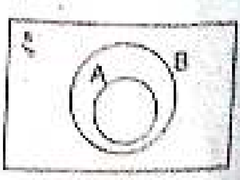


Fig. 1.10.

If necessary, we mark individual elements as points inside the diagram. Sometimes points are not marked, only elements are written inside the diagram.

Thus, the set A of factors of 12 *i.e.* $\{1, 2, 3, 4, 6, 12\}$ can be represented by the adjoining Venn diagram.

Here, $\xi = \{1, 2, 3, ..., 12\}$.

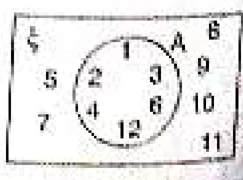


Fig. 1.11.

1.4 OPERATIONS ON SETS

Given two sets, we can combine them to form another set (which may be equal to any one of the two given sets). Of course, whatever we do with two sets can be extended to any number of sets.

1.4.1 Union of two sets

*The **union** of two sets A and B, written as A ∪ B (read as 'A union B'), is the set consisting of all the elements which belong to A or B or both. Thus*

$A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Note that $x \in A$ or $x \in B$ also includes the possibility of x belonging to both A and B. Further, if x belongs to both A and B, it should be taken as the member of the union set only once, because we know that the repetition of the members of a set does not alter it.

For example :

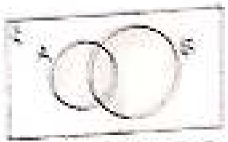
(*i*) Let A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9}, then A ∪ B = {1, 2, 3, 4, 5, 7, 9}.

(*ii*) Let A = {*a*, *b*, *c*, *d*, *e*, *i*} and B = {*c*, *a*, *i*, *d*},

then A ∪ B = {*a*, *b*, *c*, *d*, *e*, *i*} = A.

We note that if B ⊂ A, then A ∪ B = A.

If A, B are sets whose members are represented by points within circles, then their *union* is represented by points within the shaded area shown in fig. 1.12.



A ∪ B (shaded area)
Fig. 1.12

1.4.2 Intersection of two sets

*The **intersection** of two sets A and B, written as A ∩ B (read as 'A intersection B'), is the set consisting of all the elements which belong to both A and B. Thus*

$A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Note that A ∩ B is the set consisting of all the common elements of A and B.

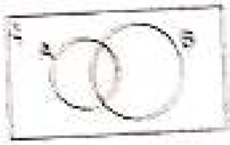
For example :

(*i*) let A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9}, then A ∩ B = {1, 3, 5}.

(*ii*) let A = {*a*, *b*, *c*, *d*, *e*, *i*} and B = {*c*, *a*, *i*, *d*}, then A ∩ B = {*a*, *c*, *d*, *i*} = B.

We note that if B ⊂ A, then A ∩ B = B.

If A, B are the sets whose members are represented by points within circles, then their *intersection* is represented by points within the shaded area shown in fig. 1.13.



A ∩ B (shaded area)
Fig. 1.13

Two sets A and B are called **disjoint** if A ∩ B = ϕ; otherwise, they are called **joint** or **overlapping** sets.

For example :

let A = {1, 3, 5} and B = {2, 4, 6, 8, 10}, then A ∩ B = ϕ.

Therefore, the sets A and B are disjoint.

Let A, B be the sets whose members are represented by points within circles and if A, B are disjoint then they can be represented by Venn diagram shown in fig. 1.14.

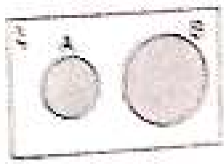


Fig. 1.14

Note that there are no elements which are common to A and B.

1.4.3 Difference of two sets

Let A, B be two sets, then A − B is the set consisting of all the elements which belong to A but do not belong to B. Thus

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.

For example :

(*i*) let A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9}, then

A − B = {2, 4} and B − A = {7, 9}.

Note that A − B ≠ B − A.

(*ii*) let A = {*b*, *c*, *d*, *e*, *o*} and B = {*a*, *e*, *i*, *o*}, then

A − B = {*b*, *c*, *d* } and B − A = {*a*, *i* }.

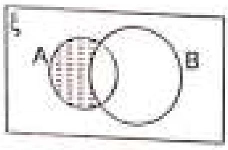
Note that A − B ≠ B − A.

(*iii*) let A = {1, 2, 3, ..., 10} and B = {3, 5, 7, 8}, then

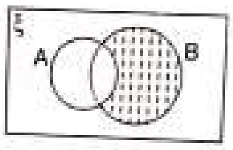
A − B = {1, 2, 4, 6, 9, 10} and B − A = ϕ.

We note that if B ⊂ A, then B − A = ϕ.

If A, B are sets whose members are represented by points within circles, then A − B and B − A are represented by points within the shaded areas shown in fig. 1.15 and fig. 1.16 respectively.



A − B (shaded area)
Fig. 1.15.



B − A (shaded area)
Fig. 1.16.

REMARK

Note that the sets A − B, A ∩ B and B − A are mutually disjoint *i.e.* the intersection of any two of these sets is the empty set as shown in fig. 1.17.

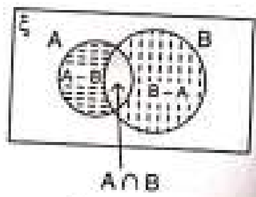


Fig. 1.17.

1.4.4 Complement of a set

Let ξ be the universal set and A be any set then the ***complement*** *of A, denoted by A′ or* A^c *or* $\overline{A}$*, is the set consisting of all the elements of ξ which do not belong to A.* Thus

A′ = {*x* : *x* ∈ ξ and *x* ∉ A}.

If the members of the set A are represented by points within the circle, then the members of A′ are represented by points within the shaded area shown in fig. 1.18.

Fig. 1.18.

Note that A′ = ξ − A.

In general, if A is any subset of a set B, then the complement of A in B is the set consisting of all the elements of B which do not belong to A.

For example :

(*i*) let ξ = {1, 2, 3, ..., 10} and A = {1, 2, 3, 5, 7, 9}, then

A′ = {4, 6, 8, 10}.

(*ii*) let ξ be the set of all the letters in the English alphabet and A be the set of all the vowels in the English alphabet, then A′ is the set of all the consonants in the English alphabet.

REMARK

If A is a subset of the universal set ξ then A′ is also a subset of ξ.

In the above example (*i*), ξ = {1, 2, 3, ..., 10} and A = {1, 2, 3, 5, 7, 9}, then

A′ = {4, 6, 8, 10}.

∴ (A′)′ = {1, 2, 3, 5, 7, 9} = A.

Thus, from the definition of the complement of a subset A of the universal set ξ it follows that (A′)′ = A.

1.4.5 Some basic results about cardinal number

If A, B are finite sets, then in counting the elements of $A \cup B$, the elements of $A \cap B$ are counted twice — once in counting the elements of A and second time in counting the elements of B. Hence

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

In particular, if $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$.

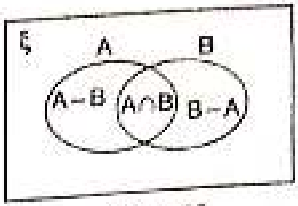


Fig. 1.19.

Also, from the Venn diagram, it is clear that

(*i*) $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$

(*ii*) $n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$

(*iii*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

(*iv*) $n(A') = n(\xi) - n(A)$, *provided ξ (the universal set) is finite.*

Further, if A, B and C are any finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C).$$

ILLUSTRATIVE EXAMPLES

Example 1. *Let* $A = \{3, 6, 9, 12, 15, 18, 21\}$ *and* $B = \{4, 8, 12, 16, 20\}$, *find*

(*i*) $A \cup B$ (*ii*) $A \cap B$ (*iii*) $A - B$ (*iv*) $B - A$.

Solution. Given $A = \{3, 6, 9, 12, 15, 18, 21\}$ and $B = \{4, 8, 12, 16, 20\}$.

(*i*) $A \cup B = \{3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21\}$

(*ii*) $A \cap B = \{12\}$

(*iii*) $A - B = \{3, 6, 9, 15, 18, 21\}$

(*iv*) $B - A = \{4, 8, 16, 20\}$.

Example 2. *Let* $A = \{x : x$ *is a positive prime less than* $10\}$ *and* $B = \{x : x \in N$ *and* $0 < x - 2 \le 6\}$, *then find*

(*i*) $A \cup B$ (*ii*) $A \cap B$ (*iii*) $A - B$ (*iv*) $B - A$.

Also verify that

(*a*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*b*) $n(A - B) = n(A \cup B) - n(B)$

(*c*) $n(B - A) = n(B) - n(A \cap B)$.

Solution. Given $A = \{x : x$ is a positive prime less than $10\} = \{2, 3, 5, 7\}$ and

$B = \{x : x \in N \text{ and } 0 < x - 2 \le 6\} = \{x : x \in N \text{ and } 2 < x \le 8\}$

$= \{3, 4, 5, 6, 7, 8\}$.

Therefore,

(*i*) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

(*ii*) $A \cap B = \{3, 5, 7\}$

(*iii*) $A - B = \{2\}$

(*iv*) $B - A = \{4, 6, 8\}$.

Further, $n(A) = 4$, $n(B) = 6$, $n(A \cup B) = 7$, $n(A \cap B) = 3$, $n(A - B) = 1$ and $n(B - A) = 3$.

Therefore,

(*a*) $n(A) + n(B) - n(A \cap B) = 4 + 6 - 3 = 7 = n(A \cup B)$

(*b*) $n(A \cup B) - n(B) = 7 - 6 = 1 = n(A - B)$ and

(*c*) $n(B) - n(A \cap B) = 6 - 3 = 3 = n(B - A)$.

Hence, all the three results (*a*), (*b*) and (*c*) are true.

Example 3. *Let X = the set of all letters in the word 'NEW DELHI' and Y = the set of all the letters in the word 'CHANDIGARH'. Find*

(i) $X \cup Y$ (ii) $X \cap Y$ (iii) $X - Y$.

Also verify that

(a) $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$ *and*

(b) $n(X - Y) = n(X \cup Y) - n(Y) = n(X) - n(X \cap Y)$.

Solution. The given sets X and Y in the roster form are

$X = \{N, E, W, D, L, H, I\}$ and $Y = \{C, H, A, N, D, I, G, R\}$

∴ (i) $X \cup Y = \{N, E, W, D, L, H, I, C, A, G, R\}$

(ii) $X \cap Y = \{N, D, H, I\}$

(iii) $X - Y = \{E, W, L\}$

Further, $n(X) = 7$, $n(Y) = 8$, $n(X \cup Y) = 11$, $n(X \cap Y) = 4$ and $n(X - Y) = 3$. Therefore,

(a) $n(X) + n(Y) - n(X \cap Y) = 7 + 8 - 4 = 11 = n(X \cup Y)$

(b) $n(X \cup Y) - n(Y) = 11 - 8 = 3 = n(X - Y)$ and

$n(X) - n(X \cap Y) = 7 - 4 = 3 = n(X - Y)$.

Hence, both the results (a) and (b) are true.

Example 4. *If $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$, then verify that*

$$L - (M \cup N) = (L - M) \cap (L - N).$$

Solution. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

then $M \cup N = \{3, 4, 5, 6, 1\}$.

∴ $L - (M \cup N) = \{1, 2, 3, 4\} - \{3, 4, 5, 6, 1\} = \{2\}$.

Now $L - M = \{1, 2, 3, 4\} - \{3, 4, 5, 6\} = \{1, 2\}$ and

$L - N = \{1, 2, 3, 4\} - \{1, 3, 5\} = \{2, 4\}$.

∴ $(L - M) \cap (L - N) = \{1, 2\} \cap \{2, 4\} = \{2\}$.

Hence, $L - (M \cup N) = (L - M) \cap (L - N)$.

Example 5. *Show that $A \cap B = A \cap C$ need not imply $B = C$.*

Solution. Let $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{1, 4\}$, then

$A \cap B = \{1\}$ and $A \cap C = \{1\}$.

Thus, $A \cap B = A \cap C$ but $B \neq C$.

Example 6. *Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.*

Solution. Take $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{2, 3\}$, then

$A \cap B = \{1\}$, $B \cap C = \{3\}$ and $A \cap C = \{2\}$.

Thus, $A \cap B$, $B \cap C$ and $A \cap C$ are all non-empty sets but $A \cap B \cap C = \phi$.

Example 7. *For all sets A, B and C, is $(A \cap B) \cup C = A \cap (B \cup C)$? Justify your answer.*

Solution. No; for example, consider the sets :

$A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$.

Then $(A \cap B) \cup C = (\{1, 2\} \cap \{2, 3\}) \cup \{3, 4\}$

$= \{2\} \cup \{3, 4\} = \{2, 3, 4\}$

and $A \cap (B \cup C) = \{1, 2\} \cap (\{2, 3\} \cup \{3, 4\})$

$= \{1, 2\} \cap \{2, 3, 4\} = \{2\}$.

Thus, $(A \cap B) \cup C \neq A \cap (B \cup C)$.

Example 8. *Taking the set of natural numbers as the universal set, write the complements of the following sets :*

(*i*) $A = \{x : x \text{ is a prime number}\}$

(*ii*) $B = \{x : x \text{ is a natural number divisible by 3 and 5}\}$

(*iii*) $C = \{x : 2x + 5 = 9\}$

(*iv*) $D = \{x : 2x + 1 > 10\}$.

Solution. Here $\xi = \mathbf{N}$. It is understood that A, B, C, D are all subsets of **N** (Universal set).

(*i*) Given $A = \{x : x \text{ is a prime number}\}$

$\therefore \quad A' = \{x : x \in \mathbf{N} \text{ and } x \text{ is not a prime number}\}$

$= \{x : x \in \mathbf{N}, x = 1 \text{ or } x \text{ is a composite number}\}$.

(*ii*) Given $B = \{x : x \text{ is a natural number divisible by 3 and 5}\}$

$= \{x : x \in \mathbf{N} \text{ and } x \text{ is divisible by 15}\}$

$\therefore \quad B' = \{x : x \in \mathbf{N} \text{ and } x \text{ is not divisible by 15}\}$.

(*iii*) Given $2x + 5 = 9 \Rightarrow x = 2 \Rightarrow C = \{2\}$

$\therefore \quad C' = \{x : x \in \mathbf{N} \text{ and } x \neq 2\}$.

(*iv*) Given $2x + 1 > 10 \Rightarrow x > \frac{9}{2}$ but $x \in \mathbf{N}$

$\Rightarrow \quad D = \{5, 6, 7, 8, \ldots\}$

$\therefore \quad D' = \{x : x \in \mathbf{N} \text{ but } x \notin D\} = \{1, 2, 3, 4\}$.

Example 9. *Let ξ be the set of all digits in our decimal system, $A = \{x : x$ is an odd integer$\}$, $B = \{x : x$ is an even integer$\}$ and $C = \{x : x \leq 5\}$, then form the following sets :*

(*i*) $(A \cup B)'$ (*ii*) $A - C$ (*iii*) $A \cap (B \cup C)$

(*iv*) $A - (B \cap C)$ (*v*) $A \cap C'$.

Solution. Here $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. It is understood that A, B and C are all subsets of ξ (the universal set).

$\therefore \quad A = \{1, 3, 5, 7, 9\}$, $B = \{0, 2, 4, 6, 8\}$ and $C = \{0, 1, 2, 3, 4, 5\}$

(*i*) $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = \xi$.

$\therefore \quad (A \cup B)' = \phi$.

(*ii*) $A - C = \{7, 9\}$.

(*iii*) $B \cup C = \{0, 1, 2, 3, 4, 5, 6, 8\}$.

$\therefore \quad A \cap (B \cup C) = \{1, 3, 5\}$.

(*iv*) $B \cap C = \{0, 2, 4\}$.

$\therefore \quad A - (B \cap C) = \{1, 3, 5, 7, 9\}$.

(*v*) $C' = \{6, 7, 8, 9\}$.

$\therefore \quad A \cap C' = \{7, 9\}$.

Example 10. *If $\xi = \{x : x \in \mathbf{N}$ and $x \leq 10\}$, $A = \{x : x$ is prime$\}$ and $B = \{x : x$ is a factor of 24$\}$, verify the following results :*

(*i*) $A - B = A \cap B'$ (*ii*) $(A \cup B)' = A' \cap B'$ (*iii*) $(A \cap B)' = A' \cup B'$.

Solution. Here $\xi = \{1, 2, 3, \ldots, 10\}$,

$A = \{2, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 6, 8\}$.

(*i*) $A - B = \{5, 7\}$, $B' = \{5, 7, 9, 10\}$

$\therefore \quad A \cap B' = \{5, 7\}$.

Hence $A - B = A \cap B'$.

(*ii*) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $\therefore \quad (A \cup B)' = \{9, 10\}$.

Also $A' = \{1, 4, 6, 8, 9, 10\}$ and $B' = \{5, 7, 9, 10\}$.

$\therefore \quad A' \cap B' = \{9, 10\}$.

Hence $(A \cup B)' = A' \cap B'$.

(iii) $A \cap B = \{2, 3\}$, $\therefore (A \cap B)' = \{1, 4, 5, 6, 7, 8, 9, 10\}$.

Also $A' = \{1, 4, 6, 8, 9, 10\}$ and $B' = \{5, 7, 9, 10\}$.

$\therefore \quad A' \cup B' = \{1, 4, 5, 6, 7, 8, 9, 10\}$.

Hence $(A \cap B)' = A' \cup B'$.

REMARK

In general, if A and B are any two subsets of the universal set ξ then the following results are true :

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

These results are called *De Morgan's Laws.*

Example 11. *If* $\xi = \{x : x \in N, x \leq 30\}$, $A = \{x : x$ *is prime* $< 5\}$, $B = \{x : x$ *is a perfect square* $\leq 10\}$ *and* $C = \{x : x$ *is a perfect cube* $\leq 30\}$, *then verify the following results :*

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(iii) $(A \cap B) \cap C = A \cap (B \cap C)$ (iv) $A' - B' = B - A$.

Solution. Given $\xi = \{x : x \in N, x \leq 30\}$, $A = \{x : x$ is prime $< 5\}$, $B = \{x : x$ is a perfect square $\leq 10\}$ and $C = \{x : x$ is a perfect cube $\leq 30\}$, then $\xi = \{1, 2, 3, \ldots, 30\}$, $A = \{2, 3\}$, $B = \{1, 4, 9\}$ and $C = \{1, 8, 27\}$.

(i) $A \cup B = \{2, 3, 1, 4, 9\} \Rightarrow (A \cup B)' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$;

$A' = \{1, 4, 5, 6, \ldots, 30\}$ and $B' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

$\Rightarrow \quad A' \cap B' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$.

$\therefore \quad (A \cup B)' = A' \cap B'$.

(ii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi \Rightarrow (A \cap B)' = \xi$;

$A' \cup B' = \{1, 4, 5, 6, \ldots, 30\} \cup \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

$= \{1, 2, 3, 4, \ldots, 30\} = \xi$

$\therefore \quad (A \cap B)' = A' \cup B'$.

(iii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi$

$\Rightarrow \quad (A \cap B) \cap C = \phi \cap C = \phi \cap \{1, 8, 27\} = \phi$

$B \cap C = \{1, 4, 9\} \cap \{1, 8, 27\} = \{1\}$

$\Rightarrow \quad A \cap (B \cap C) = \{2, 3\} \cap \{1\} = \phi$.

$\therefore \quad (A \cap B) \cap C = A \cap (B \cap C)$.

(iv) $A' = \{1, 4, 5, 6, \ldots, 30\}$ and $B' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$,

$\therefore \quad A' - B' = \{1, 4, 9\}$.

Also $B - A = \{1, 4, 9\} - \{2, 3\} = \{1, 4, 9\}$.

$\therefore \quad A' - B' = B - A$.

Example 12. *Let* $\xi = \{x : x$ *is a letter in 'AN EXCELLENT BOOK'\}*, $P = \{x : x$ *is a letter in the word 'TALENT'\} and* $Q = \{x : x$ *is a letter in the word 'BANANA'\}. Draw a Venn diagram and find the following :*

(i) $P \cup Q$ (ii) $P \cap Q$ (iii) $P - Q$ (iv) $(P \cup Q)'$.

Also verify that

(a) $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ *and*

(b) $n(P - Q) = n(P \cup Q) - n(Q) = n(P) - n(P \cap Q)$.

Solution. Given $\xi = \{x : x$ is a letter in 'AN EXCELLENT BOOK'\},

$P = \{x : x$ is a letter in word 'TALENT'\} and

$Q = \{x : x$ is a letter in word 'BANANA'\}

The sets ξ, P and Q in the roster form are :

$\xi = \{A, N, E, X, C, L, T, B, O, K\}$, $P = \{T, A, L, E, N\}$ and $Q = \{B, A, N\}$.

The Venn diagram is shown in the adjoining figure.

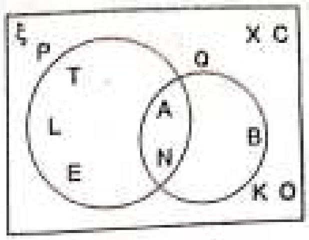


Fig. 1.20.

From Venn diagram, we have

(*i*) P ∪ Q = {T, A, L, E, N, B}

(*ii*) P ∩ Q = {A, N}

(*iii*) P − Q = {T, L, E}

(*iv*) (P ∪ Q)′ = {X, C, K, O}

Further, we find that

n (P) = 5, n (Q) = 3, n (P ∪ Q) = 6, n (P ∩ Q) = 2

and n (P − Q) = 3.

(*a*) n (P) + n (Q) − n (P ∩ Q) = 5 + 3 − 2 = 6 = n (P ∪ Q)

Hence, n (P ∪ Q) = n (P) + n (Q) − n (P ∩ Q)

(*b*) n (P ∪ Q) − n (Q) = 6 − 3 = 3 = n (P − Q);

n (P) − n (P ∩ Q) = 5 − 2 = 3 = n (P − Q).

Hence, n (P − Q) = n (P ∪ Q) − n (Q) = n (P) − n (P ∩ Q).

Example 13. *On the real axis, if A = [0, 3] and B = [2, 6], then find the following :*

(*i*) *A′* (*ii*) *B′* (*iii*) *A ∪ B* (*iv*) *A ∩ B* (*v*) *A − B* (*vi*) *B − A.*

Solution. Given A = [0, 3] = {x : x ∈ **R**, 0 ≤ x ≤ 3}

and B = [2, 6] = {x : x ∈ **R**, 2 ≤ x ≤ 6}.

The sets A and B on the real axis have been represented below :

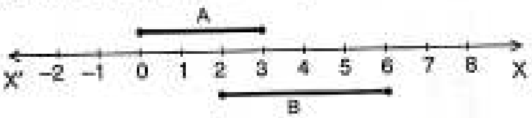


Fig. 1.21.

Using the figure :

(*i*) A′ = {x : x ∈ **R**, x ∉ A} = (−∞, 0) ∪ (3, ∞)

(*ii*) B′ = {x : x ∈ **R**, x ∉ B} = (−∞, 2) ∪ (6, ∞)

(*iii*) A ∪ B = {x : x ∈ **R**, x ∈ A or x ∈ B} = [0, 6]

(*iv*) A ∩ B = {x : x ∈ **R**, x ∈ A and x ∈ B} = [2, 3]

(*v*) A − B = {x : x ∈ **R**, x ∈ A but x ∉ B} = [0, 2)

(*vi*) B − A = {x : x ∈ **R**, x ∈ B but x ∉ A} = (3, 6].

EXERCISE 1.3

***Very short answer type questions* (1 to 17) :**

1. If A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8}, find

(*i*) A ∪ B (*ii*) A ∩ B (*iii*) A − B (*iv*) B − A.

2. If A = {x : x is a natural number and 1 < x ≤ 6} and

B = {x : x is a natural number and 6 < x < 10}, find

(*i*) A ∪ B (*ii*) A ∩ B (*iii*) A − B (*iv*) B − A.

3. Which of the following pairs of sets are disjoint?

(*i*) {*a*, *e*, *i*, *o*, *u*} and {*c*, *d*, *e*, *f*}

(*ii*) {2, 6, 10, 14, 18} and {3, 7, 11, 15}

(*iii*) {1, 2, 3, 4} and {x : x is a natural number and 4 ≤ x ≤ 6}}

(*iv*) {x : x is an even integer} and {x : x is an odd integer}.

4. If ξ = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3, 5, 6} and B = {2, 3, 4, 7, 8}, find the following :

(*i*) A ∪ B (*ii*) A ∩ B (*iii*) A − B (*iv*) B − A (*v*) A ∩ B′

(*vi*) (A ∪ B)′ (*vii*) A′ ∩ B′ (*viii*) A′ ∪ B (*ix*) (A − B)′.

5. If A and B are two sets such that $A \subset B$, then what is

(i) $A \cup B$? (ii) $A \cap B$?

6. Taking the set of natural numbers as the universal set, write the complements of the following sets :

(i) $A = \{x : x$ is an odd natural number$\}$ (ii) $B = \{x : x$ is an even natural number$\}$

(iii) $C = \{x : x$ is a multiple of $5\}$ (iv) $D = \{x : x$ is divisible by 2 and $3\}$

(v) $E = \{x : x$ is a perfect cube$\}$ (vi) $F = \{x : x + 5 = 8\}$

(vii) $G = \{x : x \geq 7\}$ (viii) $H = \{x : 3x - 1 < 14\}$.

7. Let ξ be set of all triangles in a plane. If A is the set of all triangles in that plane with atleast one angle different from 60°, what is A'?

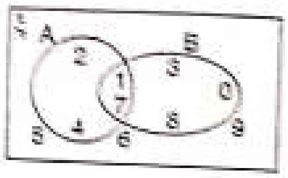
8. From the adjoining Venn diagram, determine the following sets :

(i) $A \cup B$

(ii) $A \cap B$

(iii) $A - B$

(iv) $(A \cap B)'$.

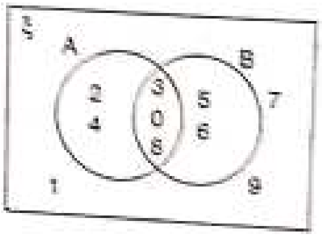


9. From the adjoining Venn diagram, write the following :

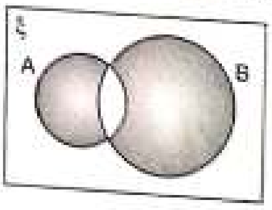
(i) A'

(ii) B'

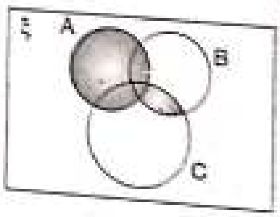
(iii) $(A \cap B)'$.



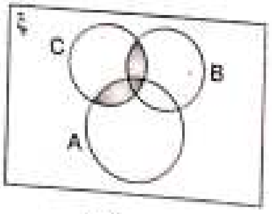
10. What is represented by the shaded regions in each of the following Venn diagrams :



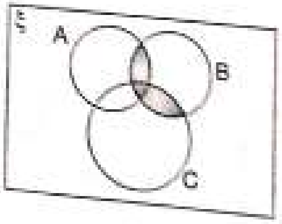
(i)



(ii)



(iii)



(iv)

11. If $\xi = \{x : x \in N, x \leq 8\}$, $A = \{x : 8 < x^2 < 40\}$ and $B = \{x : x$ is an odd integer$\}$, then draw a Venn-diagram to show the relationship between the given sets.

12. Let $\xi = \{x : x \in N$ and $x < 10\}$, $A = \{x : x = 2y + 1$ and $y \in N\}$ and $B = \{x : x = 3y - 1$ and $y \in N\}$, list the elements of $A' \cup B$.

13. If $\xi = \{1, 2, 3, ..., 10\}$, $A = \{x : x$ is prime$\}$ and $B = \{x : x$ is even integer$\}$, then write the following :

(i) $A - B$ (ii) $A \cap B'$.

14. If $\xi = \{$all digits in our decimal system$\}$, $A = \{x : x$ is prime$\}$, and $B = \{x : x^2 < 25\}$, then write the following :

(i) $B - A$ (ii) $A \cup B$ (iii) $(A \cup B)'$

15. Give an example of three sets A, B and C such that $A \neq B$ but $A - C = B - C$.

16. On the real axis, if $A = (0, 3]$ and $B = [2, 6]$, then write the following :

(*i*) $A \cup B$ (*ii*) $A \cap B$ (*iii*) $A - B$ (*iv*) $B - A$.

17. On the real axis, if $A = [-1, 1)$ and $B = [0, 4]$, then find the following :

(*i*) A' (*ii*) B' (*iii*) $A \cup B$ (*iv*) $A \cap B$ (*v*) $A - B$ (*vi*) $B - A$.

18. If A = the set of letters in the word 'JAIPUR' and B = the set of letters in the word 'JODHPUR', find the following :

(*i*) $A \cup B$ (*ii*) $A \cap B$ (*iii*) $A - B$ (*iv*) $B - A$.

Also verify the following results :

(*a*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*b*) $n(A - B) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$.

19. If $\xi = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 5, 7\}$, $B = \{0, 2, 3, 7\}$ and $C = \{0, 3, 4, 6\}$, form the following sets :

(*i*) $(A \cup B)'$ (*ii*) $A - C$ (*iii*) $A \cap (B \cup C)$.

20. If $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{4, 6, 8\}$, determine each of the following set :

(*i*) $A \cup C$ (*ii*) $A \cup B$ (*iii*) $B \cap C$ (*iv*) $A \cap \phi$

(*v*) $A \cup A'$ (*vi*) $A \cap A'$ (*vii*) $A' \cup C$ (*viii*) $A' \cup C'$

(*ix*) $B' \cap C'$ (*x*) $(A \cup B) \cap C$ (*xi*) $(A \cap B) \cap C'$ (*xii*) $(A \cup B) \cap (A \cup C)$.

21. Let $A = \{2x : x \in N \text{ and } 1 \leq x < 4\}$, $B = \{x + 2 : x \in N \text{ and } 2 \leq x < 5\}$ and $C = \{x : x \in N \text{ and } 3 < x < 6\}$, determine the following :

(*i*) $A \cap B$ (*ii*) $A \cup B$ (*iii*) $(A \cup B) \cap C$.

Also verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

22. Let $A = \{x : x \in N \text{ and } x \text{ is a multiple of } 2\}$,
$B = \{x : x \in N \text{ and } x \text{ is a multiple of } 5\}$ and
$C = \{x : x \in N \text{ and } x \text{ is a multiple of } 10\}$.

Write the set $A \cap (B \cup C)$.

23. If ξ = the set of all digits in our decimal system, $A = \{x : x \text{ is prime}\}$ and $B = \{x : x \text{ is a perfect square}\}$, then verify the following results :

(*i*) $A \cup A' = \xi$ (*ii*) $A \cap A' = \phi$ (*iii*) $A - B = A \cap B'$

(*iv*) $(A \cap B)' = A' \cup B'$ (*v*) $(A \cup B)' = A' \cap B'$ (*vi*) $(A')' = A$.

24. Let $\xi = \{x : x \in N \text{ and } x \leq 8\}$, $A = \{x : 5 < x^2 < 50\}$ and $B = \{x : x \text{ is prime}\}$.

Draw a Venn diagram to show the relationship between the given sets. Hence, list the elements of the following sets :

(*i*) A' (*ii*) B' (*iii*) $A - B$. Is $A - B = A \cap B'$?

1.5 PRACTICAL PROBLEMS ON SETS

We give below some applications of sets in solving daily life practical problems.

ILLUSTRATIVE EXAMPLES

Example 1. *If A and B are two sets such that $A \cup B$ has 18 elements, A has 8 elements and B has 15 elements, how many elements does $A \cap B$ have?*

Solution. Given $n(A) = 8$, $n(B) = 15$ and $n(A \cup B) = 18$.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 18 = 8 + 15 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 8 + 15 - 18 = 5$.

Hence, $A \cap B$ has 5 elements.

Example 2. *If A and B are two sets such that A has 40 elements, A ∪ B has 60 elements and A ∩ B has 10 elements, how many elements does B have?*

Solution. Given $n(A) = 40$, $n(A \cup B) = 60$ and $n(A \cap B) = 10$.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 60 = 40 + n(B) - 10$

$\Rightarrow \quad n(B) = 60 - 40 + 10 = 30.$

Hence, B has 30 elements.

Example 3. *If A and B are two sets such that n(A) = 17, n(B) = 23 and n(A ∪ B) = 38, find :*

(*i*) $n(A \cap B)$ (*ii*) $n(A - B)$ (*iii*) $n(B - A)$.

Solution. Given $n(A) = 17$, $n(B) = 23$ and $n(A \cup B) = 38$.

(*i*) We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 38 = 17 + 23 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 17 + 23 - 38 = 2.$

(*ii*) We know that

$n(A - B) = n(A \cup B) - n(B) = 38 - 23 = 15.$

(*iii*) We know that

$n(B - A) = n(A \cup B) - n(A) = 38 - 17 = 21.$

Example 4. *If n (A − B) = 10, n (B − A) = 8 and n (A ∩ B) = 3, find*

(*i*) $n(A \cup B)$ (*ii*) $n(A)$ (*iii*) $n(B)$.

Solution. (*i*) We know that

$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$\Rightarrow \quad n(A \cup B) = 10 + 8 + 3 = 21.$

(*ii*) We know that $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow \quad 10 = n(A) - 3 \Rightarrow n(A) = 13.$

(*iii*) We know that $n(B - A) = n(B) - n(A \cap B)$

$\Rightarrow \quad 8 = n(B) - 3 \Rightarrow n(B) = 11.$

Example 5. *If n (ξ) = 50, n (A) = 30, n (A ∩ B) = 12 and n ((A ∪ B)′) = 15, then find*

(*i*) $n(B)$ (*ii*) $n(B - A)$.

Solution. (*i*) Given $n(\xi) = 50$ and $n((A \cup B)') = 15$

$\therefore \quad n(A \cup B) = 50 - 15$

(∵ We know that $n(A') = n(\xi) - n(A) \Rightarrow n(A) = n(\xi) - n(A')$)

$\Rightarrow \quad n(A \cup B) = 35.$

But $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$\therefore \quad 35 = 30 + n(B) - 12$

$\Rightarrow \quad n(B) = 35 - 30 + 12 = 17.$

(*ii*) We know that $n(B - A) = n(B) - n(A \cap B)$

$\Rightarrow \quad n(B - A) = 17 - 12 = 5.$

Example 6. *If n (ξ) = 40 and n((A ∪ B)) = 31, then find n(A′ ∩ B′).*

Solution. Given $n(\xi) = 40$ and $n(A \cup B) = 31$.

$\therefore \quad n((A \cup B)') = n(\xi) - n(A \cup B)$

$= 40 - 31 = 9$ (∵ We know that $n(A') = n(\xi) - n(A)$)

But by De Morgan's law, we have

$A' \cap B' = (A \cup B)'$ ...(*i*)

$\therefore \quad n(A' \cap B') = n((A \cup B)') = 9$ (using (*i*))

Hence, $n(A' \cap B') = 9$.

Example 7. *If $n(\xi) = 40$, $n(A) = 25$ and $n(B) = 20$, then find*

(i) the greatest value of $n(A \cup B)$ *(ii) the least value of $n(A \cap B)$.*

Solution. (*i*) It is understood that every set under consideration is a subset of ξ (the universal set).

$\therefore$ $A \cup B \subset \xi \Rightarrow n(A \cup B) \leq n(\xi)$

$\Rightarrow$ $n(A \cup B) \leq 40$ ($\because$ $n(\xi) = 40$ given)

$\therefore$ The greatest value of $n(A \cup B) = 40$.

(*ii*) We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

But $n(A \cup B) \leq 40 \Rightarrow n(A) + n(B) - n(A \cap B) \leq 40$

$\Rightarrow$ $25 + 20 - n(A \cap B) \leq 40$

$\Rightarrow$ $45 - 40 \leq n(A \cap B)$

$\Rightarrow$ $5 \leq n(A \cap B) \Rightarrow n(A \cap B) \geq 5$.

$\therefore$ The least value of $n(A \cap B) = 5$.

Example 8. *In a school, there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach Physics and Mathematics. How many teach Physics?*

Solution. Let M be the set of teachers who teach Mathematics and P be the set of teachers who teach Physics.

We are given that :

$n(M \cup P) = 20$, $n(M) = 12$ and $n(M \cap P) = 4$.

We have to find $n(P)$.

We know that

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$\Rightarrow$ $20 = 12 + n(P) - 4$

$\Rightarrow$ $n(P) = 20 - 12 + 4 = 12$.

Hence, 12 teachers teach Physics.

Example 9. *In a group of 70 people, 52 like soft drinks and 37 like alcohol and each person likes atleast one of the two drinks. How many people like both the drinks? Justify that alcohol is injurious to health.* **(*Value Based*)**

Solution. Let S be the set of people who like soft drinks and A be the set of people who like alcohol, then $n(S) = 52$ and $n(A) = 37$.

As each person likes atleast one of the two drinks, $n(S \cup A) = 70$.

We know that

$n(S \cup A) = n(S) + n(A) - n(S \cap A)$

$\Rightarrow$ $70 = 52 + 37 - n(S \cap A)$

$\Rightarrow$ $n(S \cap A) = 52 + 37 - 70 = 19$.

Hence, 19 persons like both the drinks.

Alcohol affects brain and causes memory loss. It creates imbalance in body and mind which leads to many mental and physical problems. Driving under the effect of alcohol causes accidents and many other unlawful activities.

Example 10. *In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?*

Solution. Let H be the set of students who know Hindi and E be the set of students who know English, then

$n(H) = 100$, $n(E) = 50$ and $n(H \cap E) = 25$.

We know that $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow$ $n(H \cup E) = 100 + 50 - 25 = 125$.

Since each of the students knows either Hindi or English,

$\therefore$ the number of students in the group $= n(H \cup E) = 125$.

Example 11. *There are 200 individuals with a skin disorder; 120 had been exposed to the chemical C_1, 50 to the chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to*

(i) *chemical C_1 but not chemical C_2* (ii) *chemical C_2 but not chemical C_1*

(iii) *chemical C_1 or chemical C_2.*

Solution. Let A be the set of individuals whose are exposed to chemical C_1 and B be the set of individuals who are exposed to chemical C_2, then

$n(A) = 120$, $n(B) = 50$ and $n(A \cap B) = 30$.

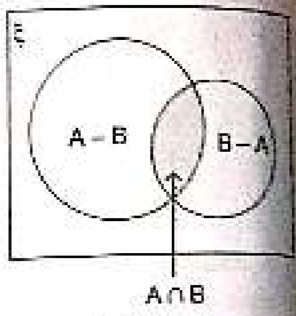


Fig. 1.22.

(i) Number of individuals exposed to chemical C_1 but not chemical C_2

$= n(A - B) = n(A) - n(A \cap B)$

$= 120 - 30 = 90.$

(ii) Number of individuals exposed to chemical C_2 but not chemical C_1

$= n(B - A) = n(B) - n(A \cap B)$

$= 50 - 30 = 20.$

(iii) Number of individuals exposed to chemical C_1 or chemical C_2

$= n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 50 - 30 = 140.$

Example 12. *In a class of 50 students, 30 students like Mathematics, 25 like Science and 16 like both. Find the number of students who like*

(i) *either Mathematics or Science* (ii) *neither Mathematics nor Science.*

Solution. We draw a Venn diagram to solve the problem.

Here ξ = all the students of the class,

M = students who like Mathematics and

S = students who like Science.

Since 16 students like both Mathematics and Science, we mark 16 in the common region of M and S. Then, as 30 students like Mathematics and of these 16 students have already been marked, therefore, 14 is marked in the remaining portion of M. Also, as 25 students like Science and of these 16 have already been marked, therefore, 9 is marked in the remaining portion of S.

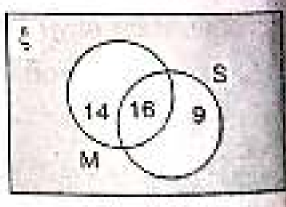


Fig. 1.23.

$\therefore$ (i) the number of students who like either Mathematics or Science

$= 14 + 16 + 9 = 39$ and

(ii) the number of students who like neither Mathematics nor Science (shown shaded in the diagram) $= 50 - 39 = 11$.

Alternatively

Here $n(\xi) = 50$, $n(M) = 30$, $n(S) = 25$ and $n(M \cap S) = 16$.

$\therefore$ (i) The number of students who like either Mathematics or Science

$= n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$= 30 + 25 - 16 = 39.$

(ii) The number of students who like neither Mathematics nor Science

$= n(M' \cap S') = n((M \cup S)')$ (De Morgan Law)

$= n(\xi) - n(M \cup S)$ $(\because n(A') = n(\xi) - n(A))$

$= 50 - 39 = 11.$

Example 13. *A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, find (i) the value of x (ii) $n(A \cup B)$.*

Solution. The adjoining Venn diagram represents the information given in the question.

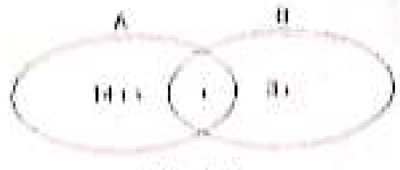


Fig. 1.24.

(*i*) From the Venn diagram, we get

$n(A) = n(A - B) + n(A \cap B)$

$= (14 + x) + x = 14 + 2x$ and

$n(B) = n(B - A) + n(A \cap B)$

$= 3x + x = 4x.$

But $n(A) = n(B)$ (given)

$\Rightarrow 14 + 2x = 4x \Rightarrow 2x = 14 \Rightarrow x = 7.$

(*ii*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$= (14 + x) + 3x + x = 14 + 5x$

$= 14 + 5 \times 7 = 14 + 35 = 49.$

Example 14. *In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. If each student has taken atleast one subject, find the number of students who have taken*

(*i*) *Biology but not Mathematics* (*ii*) *both Mathematics and Biology.*

Solution. Let M be the set of students who have taken Mathematics and B be the set of students who have taken Biology. We are given that :

$n(M \cup B) = 25$, $n(M) = 12$ and $n(M - B) = 8$.

(*i*) $n(B - M) = n(M \cup B) - n(M)$

$= 25 - 12 = 13.$

$\therefore$ The number of students who have taken Biology but not Mathematics = 13.

(*ii*) $n(M \cup B) = n(M - B) + n(B - M) + n(M \cap B)$

$\Rightarrow 25 = 8 + 13 + n(M \cap B)$

$\Rightarrow n(M \cap B) = 25 - 8 - 13 = 4.$

$\therefore$ The number of students who have taken both Mathematics and Biology = 4.

Example 15. *In class XI of a certain school, there are 20 students in a chemistry class and 30 students in a physics class. Find the number of students which are either in chemistry class or in physics class in the following cases :*

(*i*) *the two classes meet at the same hour.*

(*ii*) *the two classes meet at different hours and 10 students study both the subjects.*

Solution. Let C and P be the sets of students in chemistry class and in physics class respectively. Given that $n(C) = 20$ and $n(P) = 30$. We are required to find $n(C \cup P)$ in both the cases.

(*i*) The two classes meet at the same hour implies that $C \cap P = \phi$.

Then $n(C \cup P) = n(C) + n(P) = 20 + 30 = 50.$

(*ii*) The two classes meet at different hours and 10 students study both the subjects implies that $n(C \cap P) = 10$.

Then $n(C \cup P) = n(C) + n(P) - n(C \cap P)$

$= 20 + 30 - 10 = 40.$

Example 16. *A market research group conducted a survey of 1000 people and reported that 720 people liked product A and 450 people liked product B. What is the least number of people that must have liked both products?*

Solution. Let ξ be the set of all people surveyed, P be the set of people who liked product A and Q be the set of people who liked product B. We are given that :

$n(\xi) = 1000$, $n(P) = 720$ and $n(Q) = 450$.

We know that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$\Rightarrow n(P \cup Q) = 720 + 450 - n(P \cap Q)$

$\Rightarrow n(P \cup Q) = 1170 - n(P \cap Q)$.

Since $P \cup Q \subset \xi$, $n(P \cup Q) \leq n(\xi)$

$\Rightarrow 1170 - n(P \cap Q) \leq 1000$

$\Rightarrow 1170 - 1000 \leq n(P \cap Q)$

$\Rightarrow n(P \cap Q) \geq 170$.

Therefore, the least value of $n(P \cap Q) = 170$.

Hence, the least number of people who liked both the products is 170.

Example 17. *Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?*

Solution. Let ξ be the set of car owners investigated, P be the set of persons who owned car A and Q be the set of persons who owned car B. Then

$n(\xi) = 500$, $n(P) = 400$, $n(Q) = 200$ and $n(P \cap Q) = 50$.

We know that

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$
$$= 400 + 200 - 50 = 550.$$

Since $P \cup Q \subset \xi$, $n(P \cup Q) \leq n(\xi)$

$\Rightarrow 550 \leq 500$, which is wrong.

Hence, the given data is incorrect.

Example 18. *In an examination, 80% students passed in Mathematics, 72% passed in Science and 13% failed in both the subjects. If 312 students passed in both the subjects, find the total number of students who appeared in the examination.*

Solution. As 80% students passed in Mathematics,

$\therefore$ the students who failed in Mathematics = 20%.

Also, 72% students passed in Science.

$\therefore$ The students who failed in Science = 28%.

Since 13% students failed in both the subjects,

$\therefore$ the students who failed in Mathematics only = 7%

and the students who failed in Science only = 15%.

$\therefore$ The students who failed in either of the two subjects or in both subjects

$$= 7\% + 15\% + 13\% = 35\%.$$

The students who passed in both the subjects = 65%.

Thus, if 65 students pass in both subjects, then the total number of students = 100,

$\therefore$ if 312 students pass in both subjects, then the total number of students

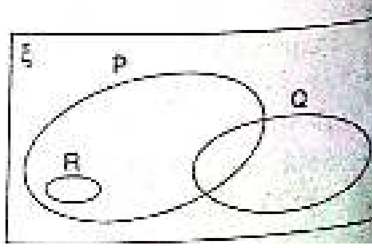
$$= \frac{100}{65} \times 312 = 480.$$

Example 19. *In the adjacent Venn diagram, if $n(\xi) = 80$, $n(P) = 40$, $n(Q) = 28$, $n(P \cap Q) = 12$ and $n(P \cap R) = 10$,*

(i) mark the number of elements in each region.

(ii) determine the value of

$n(P \cup Q)$ and $n((Q \cup R)')$.



Solution. (*i*) The number of elements in different regions are shown in the adjoining figure.

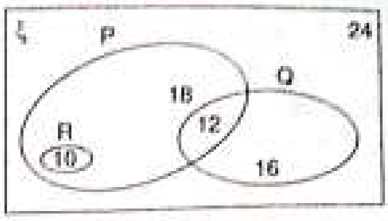


Fig. 1.26.

(*ii*) From the Venn diagram, we get

$n(P \cup Q) = 10 + 18 + 12 + 16 = 56,$

$n(Q \cup R) = 16 + 12 + 10 = 38$

$\Rightarrow n((Q \cup R)') = 80 - 38 = 42.$

Example 20. *In a survey of 60 people, it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked product C and A, 14 people liked product B and C and 8 people liked all the three products, find*

(*i*) *how many people liked product C only?*

(*ii*) *how many people like atleast one of the three products?*

(*iii*) *how many people do not like any of the three products?*

Solution. Let P, Q and R be the sets of people who like product A, B and C respectively.

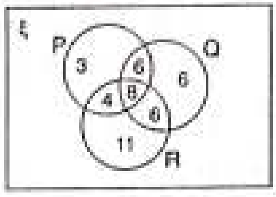


Fig. 1.27.

Given $n(P) = 21$, $n(Q) = 26$, $n(R) = 29$, $n(P \cap Q) = 14$, $n(Q \cap R) = 14$, $n(P \cap R) = 12$ and $n(P \cap Q \cap R) = 8$.

The adjoining Venn diagram represents the number of people in different regions.

From Venn diagram,

(*i*) the number of people who like product C only = 11.

(*ii*) the number of people who like atleast one of the three products

$= 3 + 6 + 8 + 4 + 6 + 6 + 11 = 44.$

(*iii*) the number of people who do not like any of the three products

$= 60 - 44 = 16.$

Example 21. *A T.V. survey gives the following data for T.V. watching : 60% watch program A; 50% watch program B; 47% watch program C; 28% watch programs A and B; 23% watch programs A and C ; 18% watch programs B and C; 8% watch programs A, B and C. Draw a Venn diagram to illustrate this information and find*

(*i*) *what percentage watch programs A and B but not C?*

(*ii*) *what percentage watch exactly two programs?*

(*iii*) *what percentage do not watch any program?*

Do you think that to some extent parents should monitor T.V. viewing habits of children? If yes, then why? **(*Value Based*)**

Solution. The adjoining Venn diagram represents the information given in the question.

From the Venn diagram, we get

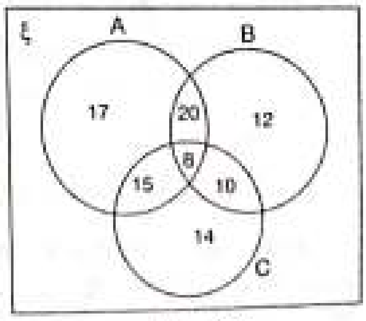


Fig. 1.28.

(*i*) the percentage of people who watch programs A and B but not C = 20%.

(*ii*) the percentage of people who watch exactly two programs

= 20% + 15% + 10% = 45%.

(*iii*) the percentage of people who watch atleast one program

= 60% + 12% + 10% + 14% = 96%,

∴ the percentage of people who do not watch any program = 4%.

Yes parents should monitor the television viewing habits of their children as there is lot of content on television which is either violent or inappropriate in nature and requires parental guidance. Children might not be able to decide which content is right or wrong for them and there the role of parents becomes important.

Example 22. *Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science; 4 in all the three. Find how many passed*

(*i*) *in English and Mathematics but not in Science*

(*ii*) *in Mathematics and Science but not in English*

(*iii*) *in Mathematics only*

(*iv*) *in more than one subject.*

Solution. Let E, M and S be the sets of students who passed in English, Mathematics and Science respectively. The adjoining Venn diagram represents the information given in the question.

Fig. 1.29.

From Venn diagram, the number of students who passed

(*i*) in English and Mathematics but not in Science = 2

(*ii*) in Mathematics and Science but not in English = 3

(*iii*) in Mathematics only = 3

(*iv*) in more than one subject = 2 + 3 + 4 = 9.

Example 23. *In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspaper, find the number of families which buy*

(*i*) *A only* (*ii*) *B only* (*iii*) *none of A, B and C.*

Solution. Let P, Q and R denote the sets of families who buy newspapers A, B and C respectively, then

$$n(P) = 40\% \text{ of } 10000 = 4000,$$
$$n(Q) = 20\% \text{ of } 10000 = 2000,$$
$$n(R) = 10\% \text{ of } 10000 = 1000,$$
$$n(P \cap Q) = 5\% \text{ of } 10000 = 500,$$
$$n(Q \cap R) = 3\% \text{ of } 10000 = 300,$$
$$n(P \cap R) = 4\% \text{ of } 10000 = 400, \text{ and}$$
$$n(P \cap Q \cap R) = 2\% \text{ of } 10000 = 200,$$

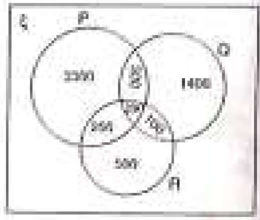


Fig. 1.30.

The adjoining Venn diagram represents the information given in the question.

From the Venn diagram, we get

(*i*) the number of families who buy newspaper A only = 3300.

(*ii*) the number of families who buy newspaper B only = 1400.

(*iii*) the number of families who buy newspaper A, B or C *i.e.* atleast one of the newspaper = 4000 + 1400 + 100 + 500 = 6000,

∴ the number of families who do not buy any of newspaper A, B and C = 10000 − 6000 = 4000.

Example 24. *In an University, out of 100 students 15 offered Mathematics only; 12 offered Statistics only; 8 offered only Physics; 40 offered Physics and Mathematics; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics. 65 offered Physics. By drawing a Venn diagram, find the number of students who*

(*i*) *offered Mathematics,*

(*ii*) *offered Statistics,*

(*iii*) *did not offer any of the above three subjects.*

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Solution. Let M, S and P be the sets of students who offered Mathematics, Statistics and Physics respectively. Let x be the number of students who offered all the three subjects, then the number of members in different regions are shown in the adjoining diagram. From the Venn diagram, we get, the number of students who offered Physics

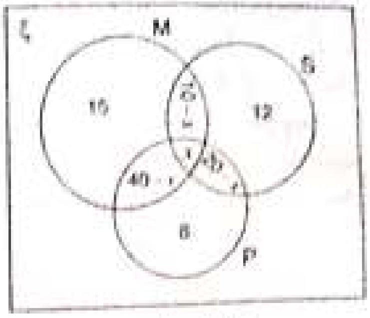

Fig. 1.31.

$= (40 - x) + x + (20 - x) + 8 = 65$ (given)

$\Rightarrow\ 68 - x = 65$

$\Rightarrow\ x = 3.$

$\therefore$ (*i*) The number of students who offered Mathematics

$= 15 + (10 - x) + x + (40 - x)$

$= 65 - x = 65 - 3 = 62.$ $(\because x = 3)$

(*ii*) The number of students who offered Statistics

$= 12 + (10 - x) + x + (20 - x)$

$= 42 - x = 42 - 3 = 39.$ $(\because x = 3)$

(*iii*) The number of students who offered any of the three subjects

$= 15 + 12 + 8 + (10 - x) + (40 - x) + (20 - x) + x$

$= 105 - 2x = 105 - 2 \times 3 = 99.$ $(\because x = 3)$

$\therefore$ The number of students who did not offer any of the three subjects $= 100 - 99 = 1.$

Example 25. *There are 240 students in class XI of a school, 130 play cricket, 100 play football, 75 play volleyball, 30 of these play cricket and football, 25 play volleyball and cricket, 15 play football and volleyball. Also each student plays atleast one of the three games. How many students play all the three games?*

Solution. Let C, F and V be the sets of students who play cricket, football and volleyball respectively.

Let x be the number of students who play all the three games, then the number of students according to given information in the question are shown in different regions in the adjoining Venn diagram.

As each student plays atleast one of three games,

$n(C \cup F \cup V) = 240.$

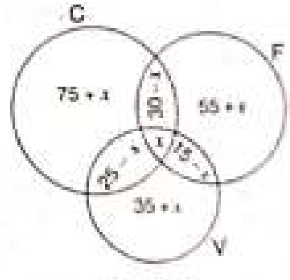

Fig. 1.32.

From the Venn diagram, we have

$(75 + x) + (30 - x) + (55 + x) + (15 - x) + (35 + x) + (25 - x) + x = 240$

$\Rightarrow\ 75 + 30 + 55 + 15 + 35 + 25 + x = 240$

$\Rightarrow\ 235 + x = 240 \Rightarrow x = 5.$

Hence, 5 students play all the three games.

Example 26. *In a survey of 100 students, the number of students studying the various languages were found to be : English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find*

(*i*) *how many students were studying Hindi?*

(*ii*) *how many students were studying English and Hindi?*

Solution. Let E, H and S be the sets of students studying English, Hindi and Sanskrit respectively; ξ be the set of students surveyed.

In the adjoining Venn diagram, let a, b, c, d, e, f and g denote the number of students in the respective regions. According to given information,

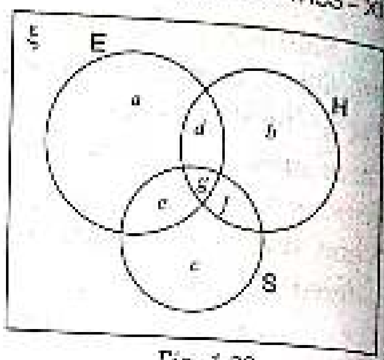

Fig. 1.33.

$n(\xi) = 100,\ a = 18,$

$a + e = 23,\ e + g = 8,$

$a + e + g + d = 26,\ e + g + f + c = 48,$

$g + f = 8$

$\Rightarrow\ a = 18,\ e = 5,\ g = 3,\ d = 0,\ f = 5,\ c = 35.$

Since 24 students study no language, therefore, the number of students who study atleast one language = 100 − 24 = 76.

$\therefore\ a + e + g + d + f + c + b = 76$

$\Rightarrow\ 18 + 5 + 3 + 0 + 5 + 35 + b = 76\ \Rightarrow b = 10.$

(*i*) The number of students studying Hindi

$= d + g + f + b = 0 + 3 + 5 + 10 = 18.$

(*ii*) The number of students studying English and Hindi

$= d + g = 0 + 3 = 3.$

Example 27. *A college awarded 38 medals for Honesty, 15 for Punctuality and 20 for Obedience. If these medals were bagged by a total of 58 students and only 3 students got medals for all three values, how many students received medals for exactly two of the three values?*

Which value you prefer to be awarded most and why? **(*Value Based*)**

Solution. Let H, P and O be the sets of students who bagged medals in Honesty, Punctuality and Obedience respectively. Then

$n(H) = 38,\ n(P) = 15,\ n(O) = 20,\ n(H \cup P \cup O) = 58$ and $n(H \cap P \cap O) = 3.$

We know that

$n(H \cup P \cup O) = n(H) + n(P) + n(O) - n(H \cap P) - n(P \cap O) - n(H \cap O) + n(H \cap P \cap O)$

$\Rightarrow\ 58 = 38 + 15 + 20 - n(H \cap P) - n(P \cap O) - n(H \cap O) + 3$

$\Rightarrow\ n(H \cap P) + n(P \cap O) + n(H \cap O) = 18$ ...(*i*)

In the adjoining Venn diagram, let x denote the number of students who got medals in Honesty and Punctuality only, y denote the number of students who got medals in Punctuality and Obedience only, z denote the number of students who got medals in Honesty and Obedience only, 3 students got medals in all the three values. Using (*i*) and Venn diagram, we get

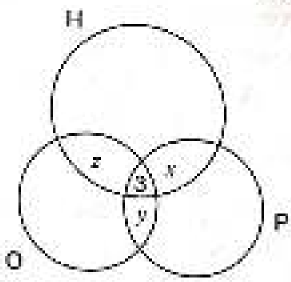

Fig. 1.34.

$(x + 3) + (y + 3) + (z + 3) = 18$

$\Rightarrow\ x + y + z = 9.$

Hence, the number of students who received medals in exactly two values out of the three values = 9.

I prefer honesty because corruption is the root cause of all problems of the country. Honest persons are always punctual, obedient and hard working. The values promoted are honesty, punctuality, obedience and hard work.

Example 28. *In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Mathematics and Chemistry and 20 none of these subjects. Find the number of students who study all the three subjects.*

Solution. Let M, P and C be the sets of students who study Mathematics, Physics and Chemistry respectively and ξ be the set of students surveyed, then $n(\xi) = 200$

It is given that the number of students who do not study any of the three subjects = 20.

∴ The number of students who study atleast one of the three subjects

$= n(\xi) - 20 = 200 - 20 = 180.$

We know that

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C)$
$\qquad - n(M \cap C) + n(M \cap P \cap C)$

$\Rightarrow 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap P \cap C)$

$\Rightarrow n(M \cap P \cap C) = 180 - 160 = 20.$

Example 29. *From 50 students taking examination in Mathematics, Physics and Chemistry, each of the student has passed in atleast one of the subjects, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry. What is the largest possible number that could have passed in all the three subjects?*

Solution: Let M, P and C be the sets of students passing in Mathematics, Physics and Chemistry respectively.

Given $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$, $n(C) = 43$,

$n(M \cap P) \le 19$, $n(M \cap C) \le 29$ and $n(P \cap C) \le 20$.

We know that

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$

$\Rightarrow \quad n(M) + n(P) + n(C) + n(M \cap P \cap C) - n(M \cup P \cup C)$
$\qquad = n(M \cap P) + n(M \cap C) + n(P \cap C)$

$\Rightarrow \quad 37 + 24 + 43 + n(M \cap P \cap C) - 50 = n(M \cap P) + n(M \cap C) + n(P \cap C)$

$\Rightarrow \quad 54 + n(M \cap P \cap C) \le 19 + 29 + 20$

$\Rightarrow \quad n(M \cap P \cap C) \le 14.$

Hence, the largest possible number of students that could have passed in all the three subjects is 14.

EXERCISE 1.4

Very short answer type questions (1 to 6) :

1. If A and B are two sets such that $n(A) = 25$, $n(B) = 32$ and $n(A \cup B) = 50$, find $n(A \cap B)$.
2. If X and Y are two sets such that X has 21 elements, Y has 32 elements and $X \cap Y$ has 11 elements, how many elements does $X \cup Y$ have?
3. If $n(A) = 20$, $n(B) = 18$ and $n(A \cap B) = 5$, calculate
   (*i*) $n(A \cup B)$ (*ii*) $n(A - B)$ (*iii*) $n(B - A)$.
4. If $n(A \cup B) = 18$, $n(A - B) = 5$, $n(B - A) = 3$, find $n(A \cap B)$.
5. If A and B are two sets such that $n(A - B) = 5$, $n(B - A) = 3$ and $n(A \cap B) = 10$, then find the following :
   (*i*) $n(A \cup B)$ (*ii*) $n(A)$ (*iii*) $n(B)$.
6. If $n(\xi) = 50$, $n(A) = 28$ and $n(B) = 30$, then what is the greatest value of $n(A \cup B)$?
7. If $n(\xi) = 48$ and $n(A' \cup B') = 36$, then find $n(A \cap B)$.
8. If $n(\xi) = 40$, $n(A) = 22$, $n(A \cap B) = 8$ and $n((A \cup B)') = 6$, determine $n(B)$.
9. If $n(\xi) = 50$, $n(A) = 20$, $n(B) = 26$ and $n((A \cup B)') = 6$, find
   (*i*) $n(A \cap B)$ (*ii*) $n(A - B)$.
10. If $n(\xi) = 50$, $n(A) = 30$ and $n(B) = 28$, find
    (*i*) the greatest value of $n(A \cup B)$ (*ii*) the least value of $n(A \cap B)$.

11. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both Hindi and English?

12. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither.

13. In a group of people, 50 people read newspaper A, 20 read newspaper B and 10 read both newspapers. How many people read atleast one of the two newspapers?

14. In a group of 65 students, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

15. In class XI of a certain school, 80 students eat burger and 42 students eat noodles in lunch time. If 24 students eat both burger and noodles, find the number of students who eat

(i) burger only (ii) noodles only (iii) any of the two food items?

Explain the importance of nutritious food over the junk food. *(Value Based)*

16. In a survey of 600 students in a school, 150 students were found to be drinking tea and 225 drinking milk and 100 students were drinking both tea and milk. How many students were drinking neither tea nor milk?

What do you think which drink should a student prefer and why? *(Value Based)*

17. In a survey of 60 people, it was found that 25 people read Newspaper H, 26 read Newspaper T, 26 read Newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all the three Newspapers. Find

(i) the number of people who read atleast one of the three Newspapers.

(ii) the number of people who read exactly one Newspaper.

18. In a survey of 100 students regarding watching T.V., it was found that 28 watch action movies, 30 watch comedy serials, 42 watch news channels, 8 watch action movies and comedy serials, 10 watch action movies and news channels, 5 watch comedy serials and news channels and 3 watch all the three programs. Draw a Venn diagram to illustrate this information and find

(i) how many watch news channels only?

(ii) how many do not watch any of the three programs?

According to you which T.V. program is useful and why? *(Value Based)*

19. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken

(i) only Chemistry (ii) only Mathematics

(iii) only Physics (iv) Physics and Chemistry but not Mathematics

(v) Mathematics and Physics but not Chemistry

(vi) only one of the subjects (vii) atleast one of the three subjects

(viii) none of the three subjects.

1.6 ALGEBRA OF SETS

We enlist some basic laws of **set theory** which are intuitively true. If A, B are any sets, then :

1. ***Idempotent Laws :***

(i) $A \cup A = A$ (ii) $A \cap A = A$.

2. ***Identity Laws :***

(i) $A \cup \phi = A$ (ii) $A \cap \xi = A$.

3. ***Boundedness Laws***

(i) $A \cup \xi = \xi$ (ii) $A \cap \phi = \phi$.

4. *Laws of complementation :*
   (i) $\xi' = \phi$ (ii) $\phi' = \xi$ (iii) $(A')' = A$
   (iv) $A \cup A' = \xi$ (v) $A \cap A' = \phi$.
5. *Commutative Laws :*
   (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$.

Now, we give some standard laws of set theory which may be visualised with the help of Venn diagrams. If A, B and C are any sets, then :

6. *Associative Laws :*
   (i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$.
7. *Distributive Laws :*
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
8. *De Morgan's Laws :*
   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.
9. (i) $A - B = A \cap B'$ (ii) $B - A = A' \cap B$.
10. $A \subset B$ if and only if $A \cap B = A$.
11. $A \subset B$ if and only if $A \cup B = B$.
12. $A \subset B$ if and only if $A - B = \phi$.
13. $A \subset B$ if and only if $B' \subset A'$.

Some of these results will be proved in the following illustrative examples.

ILLUSTRATIVE EXAMPLES

Example 1. *Let A, B and C be any sets, by drawing Venn diagram verify that*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Solution. Let A, B and C be the sets whose members are represented by regions inside the circles. In each of the diagram drawn below, the shaded region represents the set given underneath the diagram.

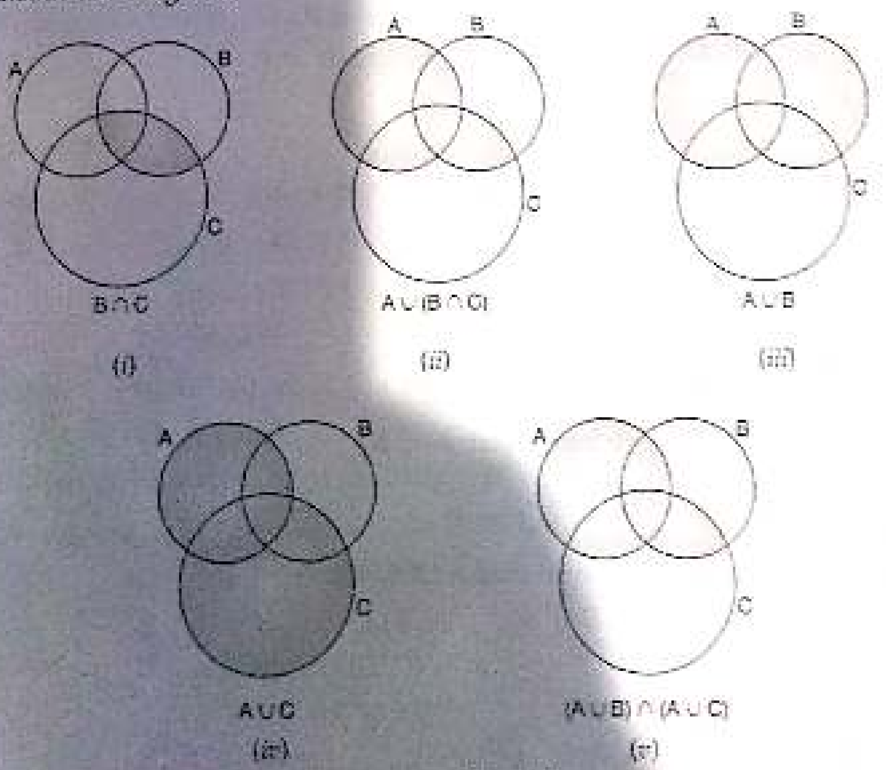

Fig. 1.35.

Since the diagrams (ii) and (v) have same shaded regions,

$\therefore$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Example 2. *If A, B are any sets, prove that* $A - B = A \cap B'$

Solution. $A - B = \{x : x \in A \text{ and } x \notin B\}$

$= \{x : x \in A \text{ and } x \in B'\}$

$= A \cap B'$

Example 3. *If A and B are any two sets, prove that*

(*i*) $A \subset A \cup B$ (*ii*) $A \cap B \subset A$.

Solution. (*i*) For all $x \in A \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$

$\Rightarrow A \subset A \cup B$

(*ii*) For all $x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A$

$\Rightarrow A \cap B \subset A$.

NOTE

Similarly, $B \subset A \cup B$ and $A \cap B \subset B$.

Example 4. *If A and B are two sets such that* $A \cup B = A \cap B$, *then prove that* $A = B$.

Solution. Given $A \cup B = A \cap B$.

We want to prove that $A = B$. For this, we shall show that $A \subset B$ and $B \subset A$.

For all $x \in A \Rightarrow x \in A \cup B$ ($\because A \subset A \cup B$, example 3)

$\Rightarrow x \in A \cap B$ ($\because A \cup B = A \cap B$, given)

$\Rightarrow x \in B$

$\Rightarrow A \subset B$.

For all $b \in B \Rightarrow b \in A \cup B$ ($\because B \subset A \cup B$)

$\Rightarrow b \in A \cap B$ ($\because A \cup B = A \cap B$, given)

$\Rightarrow b \in A$

$\Rightarrow B \subset A$.

Thus, $A \subset B$ and $B \subset A$, therefore, $A = B$.

Example 5. *If A and B are sets, prove that*

(*i*) $A \cap B = A$ *iff* $A \subset B$ (*ii*) $A \cup B = B$ *iff* $A \subset B$.

Solution. (*i*) First, let $A \cap B = A$, we want to prove that $A \subset B$.

Since $A \cap B \subset B$ therefore,

$A \subset B$ ($\because A \cap B = A$, given)

Conversely, let $A \subset B$, we want to prove that $A \cap B = A$

For all $x \in A \Rightarrow x \in A$ and $x \in B$ ($\because A \subset B$)

$\Rightarrow x \in A \cap B$

$\Rightarrow A \subset A \cap B$

Also, we know that $A \cap B \subset A$

Thus, $A \subset A \cap B$ and $A \cap B \subset A \Rightarrow A \cap B = A$.

(*ii*) First, let $A \cup B = B$, we want to prove that $A \subset B$.

Since $A \subset A \cup B$, therefore,

$A \subset B$ ($\because A \cup B = B$, given)

Conversely, let $A \subset B$, we want to prove that $A \cup B = B$.

For all $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in B$ ($\because A \subset B$)

$\Rightarrow x \in B$

$\Rightarrow A \cup B \subset B$

Also, we know that $B \subset A \cup B$

Thus, $A \cup B \subset B$ and $B \subset A \cup B \Rightarrow A \cup B = B$.

Example 6. *If $A \subset B$, then show that $C - B \subset C - A$.*

Solution. For all $x \in (C - B) \Rightarrow x \in C$ and $x \notin B$

$\Rightarrow \quad x \in C$ and $x \notin A$ $\quad (\because A \subset B$, given)

$\Rightarrow \quad x \in (C - A)$

$\Rightarrow \quad C - B \subset C - A.$

Example 7. *For all sets A, B and C, show that if*

(i) $A \subset B$ then $A \cap C \subset B \cap C$

(ii) $A \subset B$ then $A \cup C \subset B \cup C$

(iii) $A \subset C$ and $B \subset C$ then $A \cup B \subset C$.

Solution. (i) For all $x \in A \cap C \Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in B$ and $x \in C \quad (\because A \subset B)$

$\Rightarrow x \in B \cap C$

$\Rightarrow \quad A \cap C \subset B \cap C.$

(ii) For all $x \in A \cup C \Rightarrow x \in A$ or $x \in C$

$\Rightarrow x \in B$ or $x \in C \quad (\because A \subset B)$

$\Rightarrow x \in B \cup C$

$\Rightarrow \quad A \cup C \subset B \cup C.$

(iii) For all $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in C$ or $x \in C \quad (\because A \subset C$ and $B \subset C)$

$\Rightarrow x \in C$

$\Rightarrow \quad A \cup B \subset C.$

Example 8. *For any sets A and B, show that*

(i) $(A \cap B) \cup (A - B) = A$ $\qquad$ (ii) $A \cup (B - A) = A \cup B$.

Solution. (i) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B') \quad (\because A - B = A \cap B')$

$= A \cap (B \cup B')$ (Distributive law)

$= A \cap \xi \quad (\because B \cup B' = \xi)$

$= A.$

(ii) $A \cup (B - A) = A \cup (B \cap A') \quad (\because B - A = B \cap A')$

$= (A \cup B) \cap (A \cup A')$ (Distributive law)

$= (A \cup B) \cap \xi \quad (\because A \cup A' = \xi)$

$= A \cup B.$

Example 9. *For any two sets A and B, prove that*

(i) $(A - B) \cup B = A \cup B$ $\qquad$ (ii) $(A - B) \cap B = \phi$.

Solution. (i) $(A - B) \cup B = (A \cap B') \cup B \quad (\because A - B = A \cap B')$

$= (A \cup B) \cap (B' \cup B)$ (Distributive law)

$= (A \cup B) \cap \xi \quad (\because B' \cup B = \xi)$

$= A \cup B$

(ii) $(A - B) \cap B = (A \cap B') \cap B \quad (\because A - B = A \cap B')$

$= A \cap (B' \cap B)$ (Associative law)

$= A \cap \phi \quad (\because B' \cap B = \phi)$

$= \phi.$

Example 10. *For all sets A and B, show that*

(i) $(A \cup B) - B = A - B$ (ii) $A - (A \cap B) = A - B$

Solution. (i) $(A \cup B) - B = (A \cup B) \cap B'$ $(\because X - Y = X \cap Y$

$= (A \cap B') \cup (B \cap B')$ (Distributive la

$= (A \cap B') \cup \phi$ $(\because B \cap B' =$

$= A \cap B' = A - B$.

(ii) $A - (A \cap B) = A \cap (A \cap B)'$ $(\because X - Y = X \cap Y$

$= A \cap (A' \cup B')$ (De Morgan la

$= (A \cap A') \cup (A \cap B')$ (Distributive la

$= \phi \cup (A \cap B')$

$= A \cap B' = A - B$.

Example 11. *For sets A, B and C using properties of sets, prove that*

(i) $(A \cup B) \cap (A \cup B') = A$ (ii) $(A \cup B) - B = A - B$

(iii) $A - (A \cap B) = A - B$ (iv) $(A \cup B) - C = (A - C) \cup (B - C)$

(v) $A - (B \cup C) = (A - B) \cap (A - C)$ (vi) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Solution. (i) By using distributive law, we get

$(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$

$= A \cup \phi$ $(B \cap B' = \phi$

$= A$

(ii) $(A \cup B) - B = (A \cup B) \cap B'$ $(\because X - Y = X \cap Y')$

$= (A \cap B') \cup (B \cap B')$ (Distributive law)

$= (A \cap B') \cup \phi$

$= A \cap B' = A - B$.

(iii) $A - (A \cap B) = A \cap (A \cap B)'$ $(\because X - Y = X \cap Y')$

$= A \cap (A' \cup B')$ (De Morgan law)

$= (A \cap A') \cup (A \cap B')$ (Distributive law)

$= \phi \cup (A \cap B')$ $(A \cap A' = \phi)$

$= A \cap B'$

$= A - B$.

(iv) $(A \cup B) - C = (A \cup B) \cap C'$ $(\because X - Y = X \cap Y')$

$= (A \cap C') \cup (B \cap C')$ (Distributive law)

$= (A - C) \cup (B - C)$

(v) $A - (B \cup C) = A \cap (B \cup C)'$ $(\because X - Y = X \cap Y')$

$= A \cap (B' \cap C')$ (De Morgan law)

$= (A \cap A) \cap (B' \cap C')$ $(A = A \cap A)$

$= (A \cap B') \cap (A \cap C')$

$= (A - B) \cap (A - C)$.

(vi) R.H.S. $= (A \cap B) - (A \cap C) = (A \cap B) \cap (A \cap C)'$ $(\because X - Y = X \cap Y')$

$= (A \cap B) \cap (A' \cup C')$ (De Morgan law)

$= ((A \cap B) \cap A') \cup ((A \cap B) \cap C')$ (Distributive law)

$= (A' \cap (A \cap B)) \cup (A \cap (B \cap C'))$

$= ((A' \cap A) \cap B) \cup (A \cap (B - C))$ $(\because X \cap Y' = X - Y)$

$= (\phi \cap B) \cup (A \cap (B - C))$

$= \phi \cup (A \cap (B - C))$

$= A \cap (B - C) = \text{LHS}$

Example 12. *For any sets A, B and C using properties of sets, prove that:*

(i) $A - (A - B) = A \cap B$ (ii) $(A - B) \cup (A - C) = A - (B \cap C)$

(iii) $(A - B) \cap (C - B) = (A \cap C) - B$ (iv) $A - (B - C) = (A - B) \cup (A \cap C)$

Solution. (i) $A - (A - B) = A - (A \cap B')$ $(\because X - Y = X \cap Y')$

$= A \cap (A \cap B')'$

$= A \cap (A' \cup (B')')$ (De Morgan law)

$= A \cap (A' \cup B)$

$= (A \cap A') \cup (A \cap B)$ (Distributive law)

$= \phi \cup (A \cap B) = A \cap B$

(ii) $(A - B) \cup (A - C) = (A \cap B') \cup (A \cap C')$

$= A \cap (B' \cup C')$ (Distributive law)

$= A \cap (B \cap C)'$ (De Morgan law)

$= A - (B \cap C)$

(iii) $(A - B) \cap (C - B) = (A \cap B') \cap (C \cap B')$ $(\because X - Y = X \cap Y')$

$= A \cap (B' \cap C) \cap B'$ (Associative law)

$= A \cap (C \cap B') \cap B'$ (Commutative law)

$= (A \cap C) \cap (B' \cap B')$

$= (A \cap C) \cap B'$ $(\because A \cap A = A)$

$= (A \cap C) - B$ $(\because X \cap Y' = X - Y)$

(iv) $A - (B - C) = A - (B \cap C')$ $(\because X - Y = X \cap Y')$

$= A \cap (B \cap C')'$ $(\because X - Y = X \cap Y')$

$= A \cap (B' \cup (C')')$ (De Morgan law)

$= A \cap (B' \cup C)$ $(\because (A')' = A)$

$= (A \cap B') \cup (A \cap C)$ (Distributive law)

$= (A - B) \cup (A \cap C)$.

Example 13. *If $A \cap B' = \phi$, then show that $A \subset B$.*

Solution. $A = A \cap \xi = A \cap (B \cup B')$ $(\because B \cup B' = \xi)$

$= (A \cap B) \cup (A \cap B')$ (Distributive law)

$= (A \cap B) \cup \phi$ $(A \cap B' = \phi,$ given)

$= A \cap B$

$\Rightarrow$ $A \subset B$ $(\because A \subset B$ iff $A \cap B = A)$

Example 14. *If $A' \cup B = \xi$, then show that $A \subset B$.*

Solution. $B = \phi \cup B = (A \cap A') \cup B$ $(\because A \cap A' = \phi)$

$= (A \cup B) \cap (A' \cup B)$ (Distributive law)

$= (A \cup B) \cap \xi$ $(A' \cup B = \xi,$ given)

$= A \cup B$

$\Rightarrow$ $A \subset B$ $(\because A \subset B$ iff $A \cup B = B)$

Example 15. *Using properties of sets, prove that*

(*i*) $A \cup (A \cap B) = A$ (*ii*) $A \cap (A \cup B) = A$.

Solution. (*i*) $A \cup (A \cap B) = (A \cap \xi) \cup (A \cap B)$ $(\because A \cap \xi = A)$

$= A \cap (\xi \cup B)$ (Distributive law)

$= A \cap \xi$ $(\xi \cup B = \xi)$

$= A$ $(A \cap \xi = A)$

(*ii*) $A \cap (A \cup B) = (A \cup \phi) \cap (A \cup B)$ $(\because A \cup \phi = A)$

$= A \cup (\phi \cap B)$ (Distributive law)

$= A \cup \phi$ $(\phi \cap B = \phi)$

$= A$ $(A \cup \phi = A)$

Example 16. *By using properties of sets, prove that* $A \subset B$ *if and only if* $B' \subset A'$.

Solution. $A \subset B$ iff $A \cap B = A$

i.e. iff $(A \cap B)' = A'$ (Taking complements)

i.e. iff $A' \cup B' = A'$ (De Morgan law)

i.e. iff $B' \subset A'$ $(\because B \subset A \text{ iff } A \cup B = A)$

Example 17. *Let A, B be two sets. Prove that* $(A - B) \cup B = A$ *if and only if* $B \subset A$.

Solution. $(A - B) \cup B = A$ iff $(A \cap B') \cup B = A$ $(\because A - B = A \cap B')$

i.e. iff $(A \cup B) \cap (B' \cup B) = A$ (Distributive law)

i.e. iff $(A \cup B) \cap \xi = A$ $(B' \cup B = \xi)$

i.e. iff $A \cup B = A$ $(X \cap \xi = X)$

i.e. iff $B \subset A$ $(\because B \subset A \text{ iff } A \cup B = A)$

Example 18. *Let A and B be sets. If* $A \cap X = B \cap X = \phi$ *and* $A \cup X = B \cup X$ *for some set X, show that* $A = B$.

Solution. Given $A \cap X = B \cap X = \phi$ ...(*i*)

and $A \cup X = B \cup X$ ...(*ii*)

$A = A \cup \phi = A \cup (B \cap X)$ (using (*i*))

$= (A \cup B) \cap (A \cup X)$ (Distributive law)

$= (A \cup B) \cap (B \cup X)$ (using (*ii*))

$= (A \cup B) \cap (X \cup B)$ $(\because A \cup B = B \cup A)$

$= (A \cap X) \cup B$ (Distributive law)

$= \phi \cup B$ (using (*i*))

$= B$.

Example 19. *Let A, B and C be three sets* $A \cup B = A \cup C$ *and* $A \cap B = A \cap C$, *show that* $B = C$.

Solution. Consider any $b \in B \Rightarrow b \in A \cup B$

$\Rightarrow b \in A \cup C$ $(\because A \cup B = A \cup C)$

$\Rightarrow b \in A$ or $b \in C$.

Two cases arise :

Case I. If $b \in C$, then $B \subset C$

Case II. If $b \in A$, then $b \in A \cap B$

$\Rightarrow b \in A \cap C$ $(\because A \cap B = A \cap C)$

$\Rightarrow b \in C \Rightarrow B \subset C$

Thus, in both cases $B \subset C$.

Similarly, $C \subset B$. Hence $B = C$

Example 20. *If A, B and C are three sets such that $A \cup B = C$ and $A \cap B = \phi$ then show that $A = C - B$.*

Solution.

$$
\begin{aligned}
C - B &= (A \cup B) - B && (\because A \cup B = C \text{ given}) \\
&= (A \cup B) \cap B' && (\because X - Y = X \cap Y') \\
&= (A \cap B') \cup (B \cap B') && \text{(Distributive law)} \\
&= (A \cap B') \cup \phi = A \cap B' \\
&= A - B \\
&= A && (\because A \cap B = \phi)
\end{aligned}
$$

Example 21. *For any two sets A and B, prove that*

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

Solution.

$$
\begin{aligned}
(A - B) \cup (B - A) &= (A \cap B') \cup (B \cap A') \\
&= ((A \cap B') \cup B) \cap ((A \cap B') \cup A') && \text{(Distributive law)} \\
&= ((A \cup B) \cap (B' \cup B)) \cap ((A \cup A') \cap (B' \cup A')) && \text{(Distributive law)} \\
&= ((A \cup B) \cap U) \cap (U \cap (B' \cup A')) \\
&= (A \cup B) \cap (B' \cup A') \\
&= (A \cup B) \cap (A' \cup B') && \text{(Commutative law)} \\
&= (A \cup B) \cap (A \cap B)' && \text{(De Morgan law)} \\
&= (A \cup B) - (A \cap B) && (X \cap Y' = X - Y)
\end{aligned}
$$

Example 22. *For any sets A, B and C, show that*

$$(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$$

Solution.

$$
\begin{aligned}
&(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' \\
&= (A \cup B \cup C) \cap (A \cap (B' \cap C'))' \cap C' \\
&= (A \cup (B \cup C)) \cap (A \cap (B \cup C)')' \cap C' && \text{(De Morgan law)} \\
&= (A \cup (B \cup C)) \cap (A' \cup (B \cup C)) \cap C' && \text{(using De Morgan law and } (X')' = X) \\
&= ((A \cap A') \cup (B \cup C)) \cap C' && \text{(Distributive law)} \\
&= (\phi \cup (B \cup C)) \cap C' \\
&= (B \cup C) \cap C' \\
&= (B \cap C') \cup (C \cap C') && \text{(Distributive law)} \\
&= (B \cap C') \cup \phi \\
&= B \cap C'
\end{aligned}
$$

Example 23. *If $A = \{1, 3, 5, \ldots, 17\}$ and $B = \{2, 4, 6, \ldots, 18\}$ and N the set of natural numbers is the universal set, then show that $A' \cup ((A \cup B) \cap B') = N$.*

Solution. Given $A = \{1, 3, 5, \ldots, 17\}$ and $B = \{2, 4, 6, \ldots, 18\}$,

$$
\begin{aligned}
\therefore \quad A \cap B &= \phi && \ldots(i) \\
\therefore \quad A' \cup ((A \cup B) \cap B') &= A' \cup ((A \cap B') \cup (B \cap B')) && \text{(Distributive law)} \\
&= A' \cup ((A \cap B') \cup \phi) && (\because B \cap B' = \phi) \\
&= A' \cup (A \cap B') && (A \cup \phi = A) \\
&= (A' \cup A) \cap (A' \cup B') && \text{(Distributive law)} \\
&= N \cap (A \cap B)' && \text{(De Morgan law)} \\
&= N \cap \phi' && \text{(using (i))} \\
&= N \cap N \\
&= N
\end{aligned}
$$

Example 24. *If P(A) = P(B), show that A = B.*

Solution. For all $a \in A \Rightarrow \{a\} \subset A \Rightarrow \{a\} \in P(A)$

$\Rightarrow \{a\} \in P(B)$ $(\because P(A) = P(B)$, given$)$

$\Rightarrow \{a\} \subset B \Rightarrow a \in B$

$\Rightarrow A \subset B$.

For all $b \in B \Rightarrow \{b\} \subset B \Rightarrow \{b\} \in P(B)$

$\Rightarrow \{b\} \in P(A)$ $(\because P(A) = P(B)$, given$)$

$\Rightarrow \{b\} \subset A \Rightarrow b \in A$

$\Rightarrow B \subset A$.

Thus, $A \subset B$ and $B \subset A \Rightarrow A = B$.

Example 25. *For any sets A and B, prove that $P(A) \cup P(B) \subset P(A \cup B)$.*

Solution. Consider any $X \in P(A) \cup P(B)$, where X is a set

$\Rightarrow X \in P(A)$ or $X \in P(B)$

$\Rightarrow X \subset A$ or $X \subset B$

$\Rightarrow X \subset A \cup B$

$\Rightarrow X \in P(A \cup B)$

$\Rightarrow P(A) \cup P(B) \subset P(A \cup B)$.

Example 26. *For any sets A and B, prove that $P(A \cap B) = P(A) \cap P(B)$.*

Solution. $P(A \cap B) = \{X : X \in P(A \cap B)\}$, where X is a set

$= \{X : X \subset A \cap B\}$

$= \{X : X \subset A$ and $X \subset B\}$

$= \{X : X \in P(A)$ and $X \in P(B)\}$

$= \{X : X \in P(A) \cap P(B)\}$

$= P(A) \cap P(B)$.

Example 27. *A and B are two finite sets such that $n(A) = m_1$ and $n(B) = m_2$, then find the least and the greatest values of $n(A \cup B)$.*

Solution. We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = m_1 + m_2 - n(A \cap B)$

$\Rightarrow n(A \cup B) \leq m_1 + m_2$ ...(i) $(\because n(A \cap B) \geq 0)$

It may be noted that the equality sign holds only when $n(A \cap B) = 0$ i.e. when $A \cap B = \phi$ i.e. when A and B are disjoint.

Also, we know that $A \subset A \cup B$ and $B \subset A \cup B$

$\Rightarrow n(A) \leq n(A \cup B)$ and $n(B) \leq n(A \cup B)$

$\Rightarrow m_1 \leq n(A \cup B)$ and $m_2 \leq n(A \cup B)$

$\Rightarrow n(A \cup B) \geq m_1$ and $n(A \cup B) \geq m_2$

$\Rightarrow n(A \cup B) \geq \max. \{m_1, m_2\}$ ...(ii)

It may be noted that the equality sign holds only when either $A \subset B$ or $B \subset A$.

From (i) and (ii), it follows that

$\max. \{m_1, m_2\} \leq n(A \cup B) \leq m_1 + m_2$.

Hence, the least value of $n(A \cup B) = \max. \{m_1, m_2\}$ and the greatest value of $n(A \cup B) = m_1 + m_2$.

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Example 28. *A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If x% of the people watch both news channels, then prove that* $39 \le x \le 63$.

Solution. Let the number of people surveyed be 100, then

$n(A) = 63,\ n(B) = 76$ and $n(A \cap B) = x$.

As $A \cup B \subset \xi$ (universal set), $n(A \cup B) \le 100$.

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow\ n(A \cup B) = 63 + 76 - x$

$\Rightarrow\ 139 - x = n(A \cup B)$

$\Rightarrow\ 139 - x \le 100$ $\quad (\because n(A \cup B) \le 100)$

$\Rightarrow\ 139 - 100 \le x \Rightarrow 39 \le x$ ...(i)

Also, we know that $A \cap B \subset A$ and $A \cap B \subset B$

$\Rightarrow\ n(A \cap B) \le n(A)$ and $n(A \cap B) \le n(B)$

$\Rightarrow\ x \le 63$ and $x \le 76 \Rightarrow x \le 63$ ...(ii)

From (i) and (ii), we get $39 \le x \le 63$.

EXERCISE 1.5

Very short answer type questions (1 to 4) :

1. State whether the following statements are true or false :
   (i) $A \cup A = A$ (ii) $A \cap A = \phi$ (iii) $A \cup A' = \xi$ (iv) $A \cap A' = \phi$
   (v) $A \subset B \Rightarrow A \cup B = B$ (vi) $A \subset B \Rightarrow A \cap B = A$
   (vii) $A \cap \phi = A$ (viii) $A \cup \phi = A$.
2. Fill in the blanks to make each of the following a true statement :
   (i) $\phi' \cap A = \ldots$ (ii) $\xi' \cap A = \ldots$
3. Show that $A \cap B = A \cap C$ need not imply $B = C$.
   Hint. Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{0, 2, 4\}$
4. Find sets A, B and C such that $A \cap B$, $A \cap C$ and $B \cap C$ are non-empty and
   $A \cap B \cap C = \phi$.
5. Let A, B and C be any sets; by drawing Venn diagram verify that
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
6. Show that the following four conditions are equivalent :
   (i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cap B = A$ (iv) $A \cup B = B$.

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ANSWERS

EXERCISE 1.1

1. (*ii*), (*iii*), (*v*), (*vii*) and (*ix*) are sets
2. (*i*), (*ii*), (*iii*) are true

   (*iv*) is not true because $9 \notin A$

   (*v*) is not true because {3} is a set and not an element

   (*vi*) is not true because {5, 7} is a set
3. (*i*) {6, 12, 18, 24, 30, 36, 42, 48} (*ii*) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}\right\}$

   (*iii*) {2, 3, 7} (*iv*) {–3, –1, 1, 3, 5, 7} (*v*) {1, 2, 3}

   (*vi*) {5, 6, 7, ...} (*vii*) {..., –1, 0, 1, 2, 3} (*viii*) {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

   (*ix*) {0, 1, 2, 3, 4} (*x*) {1, 2, 3, 4} (*xi*) {1, 2, 3, 4, 5, 6}

   (*xii*) {–3, –2, –1, 0, 1, 2, 3} (*xiii*) {0, 1, 2, ..., 9}

   (*xiv*) {T, R, I, G, O, N, M, E, Y} (*xv*) {*a*, *e*, *i*, *o*}

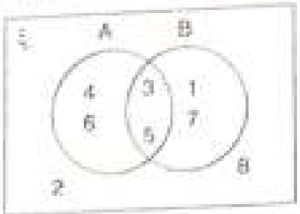
   (*xvi*) {*b*, *c*, *d*, *f*, *g*, *h*, *j*}

4. (i) {x : x is an odd natural number less than 14}
(ii) {x : x is an even natural number} (iii) {x : x = 3n, n ∈ **N** and n ≤ 5}
(iv) {x : x = 2^n, n ∈ **N** and n ≤ 6} (v) {x : x = 5^n : n ∈ **N** and n ≤ 4}
(vi) {x : x = n^2, n ∈ **N** and n ≤ 10} (vii) {x : x = n^2, n ∈ **N**}
(viii) $\left\{x : x = \frac{n}{n+1}, n \in \mathbf{N}\right\}$
5. (ii), (iii), (iv), (v), (vi)
6. (i) finite (ii) infinite (iii) finite (iv) finite (v) infinite (vi) finite
7. (i) 0 (ii) 1 (iii) 3 (iv) 0 (v) 7 (vi) 2
8. (i) 8 (ii) 8 (iii) 8
9. (i) matches (b), (ii) matches (e), (iii) matches (d), (iv) matches (a), (v) matches (c)
10. (i) False (ii) True (iii) True (iv) True
11. (i) infinite (ii) infinite (iii) finite; 0 (iv) finite; 12 (v) finite; 7
(vi) finite; 6 (vii) finite; 3 (viii) infinite (ix) finite, 11 (x) finite; 14

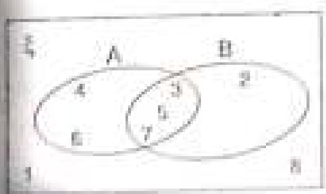
EXERCISE 1.2

1. (i) A = B (ii) A ≠ B (iii) A ≠ B (iv) A ≠ B (v) A ≠ B (vi) A = B
(vii) A = B (viii) A ≠ B
2. (i) False (ii) False (iii) True (iv) False (v) True (vi) False
(vii) False (viii) True (ix) False (x) True (xi) False
3. (i) ∈ (ii) ∉ (iii) ⊂ (iv) = (v) = (vi) ⊃ (vii) ∈
4. (i) False (ii) False (iii) True (iv) False (v) True (vi) True
5. (i) False (ii) False (iii) False (iv) False (v) False (vi) True
6. (i), (v), (vi), (viii), (ix) and (xi)
7. A = {2, 3}
8. (i) φ, {a} (ii) φ, {a}, {b}, {a, b} (iii) φ
9. {φ, {−1}, {0}, {2}, {−1, 0}, {−1, 2}, {0, 2}, {−1, 0, 2}}
10. 31 **11.** 7 **12.** 4
13. (i) (−3, 5] (ii) (−5, −1) (iii) [2, 7] (iv) [0, 3)
(v) (−∞, 5] (vi) (−∞, −3) (vii) [−2, ∞)
14. (i) {x : x ∈ **R**, −2 < x ≤ 0} (ii) {x : x ∈ **R**, 2 < x < 7}
(iii) {x : x ∈ **R**, −5 ≤ x ≤ −2} (iv) {x : x ∈ **R**, −9 ≤ x < 4}
(v) {x : x ∈ **R**, x > 3} (vi) {x : x ∈ **R**, x ≤ −1}
(vii) {x : x ∈ **R**, x < 4}
15. A = {1, 2, 3, 4, 5, 6, 7} and B = {1, 2, 3, 4, 5, 6}
16. (i) A = {0, 6, 12, 18, 24, 30, 36, 42, 48}, B = {0, 7, 14, 21, 28, 35, 42, 49}
and C = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36}
(ii) 9; 8; 11
17. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
18. (i) A = {A, C, U, M, L, T, O, R} (ii) 3
(iii) {φ, {A}, {U}, {O}, {A, U}, {A, O}, {U, O}, {A, U, O}}
19. φ, {0}, {1}, {2}, {3}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {0, 1, 2}, {0, 1, 3}, {0, 2, 3},
{1, 2, 3}
20. 32; 63 **21.** m = 7, k = 4

EXERCISE 1.3

1. (i) {1, 2, 3, 4, 5, 6, 8} (ii) {2, 4, 6} (iii) {1, 3, 5} (iv) {8}
2. (i) {2, 3, 4, 5, 6, 7, 8, 9} (ii) φ (iii) A (iv) B
3. (ii) and (iv)
4. (i) ξ (ii) {2, 3} (iii) {1, 5, 6} (iv) {4, 7, 8} (v) {1, 5, 6}
   (vi) φ (vii) φ (viii) {2, 3, 4, 7, 8} (ix) {2, 3, 4, 7, 8}
5. (i) B (ii) A
6. (i) A' = {x : x is an even natural number}
   (ii) B' = {x : x is an odd natural number}
   (iii) C' = {x : x ∈ N and x is not a multiple of 5}
   (iv) D' = {x : x ∈ N and x is not divisible by 6}
   (v) E' = {x : x ∈ N and x is not a perfect cube}
   (vi) F' = {x : x ∈ N and x ≠ 3}
   (vii) G' = {1, 2, 3, 4, 5, 6}
   (viii) H' = {x : x ∈ N and x ≥ 5}
7. The set of all equilateral triangles
8. (i) {0, 1, 2, 3, 4, 7, 8} (ii) {1, 7} (iii) {2, 4} (iv) {0, 2, 3, 4, 5, 6, 8, 9}
9. (i) {1, 5, 6, 7, 9} (ii) {1, 2, 4, 7, 9} (iii) {1, 2, 4, 5, 6, 7, 9}
10. (i) (A – B) ∪ (B – A) (ii) A ∪ (B ∩ C) (iv) (A ∩ B) ∪ (C ∩ B) or (A ∪ C) ∩ B
    (iii) (A ∩ C) ∪ (B ∩ C) or (A ∪ B) ∩ C
11. 

12. {1, 2, 4, 6, 8, 5}
13. (i) {3, 5, 7} (ii) {3, 5, 7}
14. (i) {0, 1, 4} (ii) {0, 1, 2, 3, 4, 5, 7} (iii) {6, 8, 9}
15. A = {1, 2}, B = {1, 3} and C = {0, 2, 3}
16. (i) (0, 6] (ii) [2, 3] (iii) (0, 2) (iv) (3, 6]
17. (i) (– ∞, –1) ∪ [1, ∞) (ii) (– ∞, 0) ∪ (4, ∞) (iii) [–1, 4]
    (iv) [0, 1) (v) [–1, 0) (vi) [1, 4]
18. (i) {I, A, L, P, U, R, O, D, H} (ii) {I, P, U, R} (iii) {A, I} (iv) {O, D, H}
19. (i) {1, 4, 6} (ii) {2, 5, 7} (iii) {2, 7}
20. (i) {1, 3, 4, 5, 6, 8} (ii) B (iii) {4} (iv) φ (v) ξ (vi) φ
    (vii) {0, 2, 4, 6, 7, 8, 9} (viii) ξ (ix) {0, 7, 9} (x) {4} (xi) A (xii) {1, 3, 4, 5}
21. (i) {4, 6} (ii) {2, 4, 5, 6} (iii) {4, 5}
22. {10, 20, 30, ...}
23. (i) {1, 2, 8}
    (ii) {1, 4, 6, 8}
    (iii) {4, 6} ; Yes



EXERCISE 1.4

1. 10
2. 42
3. (i) 33 (ii) 15 (iii) 13
4. 10
5. (i) 18 (ii) 15 (iii) 13
6. 50
7. 12
8. 20

9. (*i*) 2 (*ii*) 18 10. (*i*) 50 (*ii*) 8 11. 50 12. 25

13. 60 14. 25; 35

15. (*i*) 26 (*ii*) 18 (*iii*) 68

Junk food is often consumed more than requirement that increases weight which causes many diseases such as diabetes, B.P. etc. Nutritious food keeps body healthy and fit. It increases working capacity and mind remains tension free.

16. 325; Students should prefer milk because it keeps body healthy and fit, provides energy and all nutrients. Milk is a complete food.

17. (*i*) 52 (*ii*) 30

18. (*i*) 30 (*ii*) 20

New channels and comedy serials.

News channels provide information about what is going on in our country at present and abroad. Comedy serials are interesting and enjoyable.

19. (*i*) 5 (*ii*) 4 (*iii*) 2 (*iv*) 1 (*v*) 6 (*vi*) 11 (*vii*) 23 (*viii*) 2

EXERCISE 1.5

1. (*i*) True (*ii*) False (*iii*) True (*iv*) True (*v*) True (*vi*) True (*vii*) False (*viii*) True

2. (*i*) A (*ii*) ϕ

4. We may take A = {1, 2}, B = {2, 3} and C = {1, 3}. Answer is not unique