Syllabus

Elementary concepts of differentiation and integration for describing motion.

In Physics, concepts of mathematics are frequently used. To make the students conversant with mathematical tools generally used in Physics, some topics have been briefly described in this chapter.

1 Quadratic Equation

An algebraic equation of second order (in which highest power of variable is 2) is called a quadratic equation. For example,

$$ax^2 + bx + c = 0 \qquad \text{where } a \neq 0$$

is a general quadratic equation, where a and b are coefficients of x^2 and x respectively and c is a constant term. Its solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

As an example, a quadratic equation is $10x^2 - 27x + 5 = 0$. Here, $a = 10$, $b = -27$ and $c = 5$. Its solution is

$$x = \frac{27 \pm \sqrt{(-27)^2 - (4 \times 10 \times 5)}}{2 \times 10}$$

$$= \frac{27 \pm \sqrt{729 - 200}}{20} = \frac{27 \pm \sqrt{529}}{20} = \frac{27 \pm 23}{20} = \frac{50}{20}, \frac{4}{20}.$$

or

$$x = \frac{5}{2}, \frac{1}{5}.$$

2 Binomial Theorem

I. The binomial theorem, for any positive integral index n, states that

$$(x + a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \ldots\ldots + {}^nC_r x^{n-r} a^r + \ldots\ldots + {}^nC_n a^n,$$

where a is constant and $\quad {}^nC_r = \dfrac{n!}{r!\,(n-r)!}.$

Total number of terms in the expansion = index (n) + 1. In this

$$n! = n(n-1)(n-2)(n-3) \ldots\ldots\ldots 4 \cdot 3 \cdot 2 \cdot 1.$$

For example,

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$4! = 4 \times 3 \times 2 \times 1 = 24, \qquad \text{etc.}$$

II. In Physics, we generally come across the expansions of the form $(1 + x)^n$ where $|x| < 1$ and n may be an integer (positive or negative) or a fraction (positive or negative). This expansion is written as

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \ldots$$

Since $|x| < 1$, the terms having higher powers of x (x^3 and higher) are generally neglected.

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3 Exponential Series and Logarithmic Series

In general, the exponential series is written as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots\ldots$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots\ldots\ldots = 2.718.$$

On this basis, we can see that :

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots\ldots\ldots$$

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots$$

and

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots\ldots$$

The logarithmic series are :

$$\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots$$

$$\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots = 0.6931$$

$$\log_e(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\ldots$$

$$\frac{1}{2}\log_e\left(\frac{1 + x}{1 - x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots$$

4 Logarithm

The logarithm of a number at a given base is the power to which the base must be raised to represent the number. For example, we have

$$100 = 10^2$$

Then $\log_{10} 100 = 2.$

Thus if $m = n^p$, then $\log_n m = p.$

This implies that, $\log_a 1 = 0$ since $a^0 = 1$

and $\log_a a = 1$ since $a^1 = a$

i.e., the logarithm of a number 'a' at the same base 'a' is always equal to 1. Therefore,

$$\log_3 3 = 1, \log_{23} 23 = 1 \text{ etc.}$$

Following formulae are commonly required in dealing with logarithms :

(i) $\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

(iii) $\log_a m^n = n \log_a m$

(iv) To change the base, $\log_a m = \log_b m \times \log_a b$

(v) $\log_a m = \dfrac{\log_b m}{\log_b a}$

In above formulae, a, b, m and n are positive integers.

vi) $\log_e m = \log_{10} m \times \log_e 10 = 2.3026 \log_{10} m$

e., to convert the base from e (Neperian log or Natural log) to base 10 (Common logarithm) multiply y 2.3026.

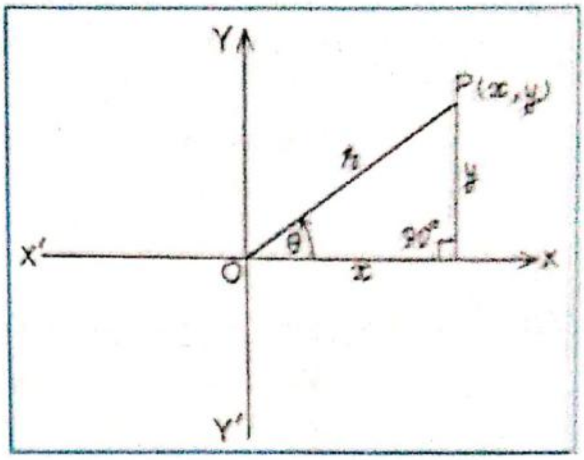
n tables the values of logarithms of numbers are given at base 10.

Trigonometric Ratios

If θ is the angle subtended by a line whose initial point is the origin and end-point is $P(x, y)$, with the X-axis then we have

$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}, \tan\theta = \frac{y}{x},$$

$$\cot\theta = \frac{x}{y}, \sec\theta = \frac{r}{x}, \text{cosec}\,\theta = \frac{r}{y}.$$



(Fig. 1)

Clearly, $\text{cosec}\,\theta = \frac{1}{\sin\theta}$,

$$\sec\theta = \frac{1}{\cos\theta} \text{ and } \cot\theta = \frac{1}{\tan\theta}.$$

From these ratios, we can see that

$$\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta \quad \text{and} \quad 1 + \cot^2\theta = \text{cosec}^2\theta$$

The angle θ is positive if it is measured anticlockwise (as shown), and negative if measured clockwise. The angle is expressed in degrees or in radians. **1 *radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.*** The relation between radian and degree is

$$\pi \text{ radian} = 180°$$

or

$$1 \text{ radian} = \frac{180°}{\pi} = \frac{180°}{3.14} = 57.3°.$$

Thus, $90° = \frac{\pi}{2}$ rad, $60° = \frac{\pi}{3}$ rad, and so on. Also

$$1° = 60' \text{ (minutes of arc) and } 1' = 60'' \text{ (seconds of arc)}.$$

Trigonometric Ratios of Few Standard Angles :

Angle θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Trigonometric Ratios of Angles : $-\theta$, $90° - \theta$, $90° + \theta$, $180° - \theta$, $180° + \theta$ ...., and $360° + \theta$ in terms of θ.

Angle θ	$-\theta$	$90° - \theta$	$90° + \theta$	$180° - \theta$	$180° + \theta$	$270° - \theta$	$270° + \theta$	$360° - \theta$	$360° + \theta$
sin	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
cos	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$

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Examples : Find the values of (i) cos (–30°), (ii) cos 210°, (iii) tan 135°, (iv) cos 150°, (v) sin 330°.

Solutions : (i) $\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$.

(ii) $\cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$.

(iii) $\tan 135° = \tan(90° + 45°) = -\cot 45° = -1$.

(iv) $\cos 150° = \cos(90° + 60°) = -\sin 60° = -\frac{\sqrt{3}}{2}$.

(v) $\sin 330° = \sin(270° + 60°) = -\cos 60° = -\frac{1}{2}$.

or $\sin 330° = \sin(360° - 30°) = -\sin 30° = -\frac{1}{2}$.

6 Few Important Trigonometric Formulae

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

(ii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

(iii) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

(iv) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$.

(v) $\sin(A-B) = \sin A \cos B - \cos A \sin B$.

(vi) $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

(vii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

(viii) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$.

7 Trigonometric Ratios of Multiple and Submultiple Angles

(i) $\sin 2A = 2 \sin A \cos A$.

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

(iii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$.

8 Product Formulae

(i) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$.

(ii) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$.

(iii) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$.

(iv) $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$.

9 Sum and Difference Formulae

(i) $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$.

(ii) $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$.

(iii) $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$.

(iv) $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$.

10 Some Important Limits

(i) $\lim_{\theta \to 0} \sin\theta = \theta$ (in radian)

(ii) $\lim_{\theta \to 0} \cos\theta = 1$

(iii) $\lim_{\theta \to 0} \tan\theta = \theta$ (in radian)

(iv) $\lim_{\theta \to 0} \tan\theta = \sin\theta = \theta$

(v) $\lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right) = 1$

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11 Elementary Concepts of Differentiation and Integration*

(A) Differentiation

If we write down a function relating two variables x and y such as

$$y = 5x + 3, \quad y = 2\sin^2 x, \quad y = e^x + 7 \text{ etc.}$$

then this indicates that if we change x then y will change depending upon the nature of the function. For example, if a boy is running then the distance covered by him will change as the time changes.

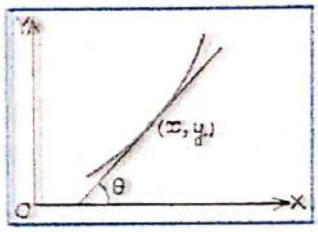
The ratio of change in dependent variable (say y), *i.e.*, Δy with change in independent variable (which we actually change, say x) *i.e.*, Δx is

$$\frac{\Delta y}{\Delta x}$$

and this depends upon the nature of function. For some functions it may be constant (independent of x and y) and for others it may depend upon either one or both the variables. When the change done in x *i.e.*, Δx is very small *i.e.*, $\Delta x \to 0$, then Δy is written as dy and the ratio becomes $\frac{dy}{dx}$. Thus

$$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right) = \frac{dy}{dx}$$

$\frac{dy}{dx}$ is called the differentiation of y with respect to x (or the differential coefficient of y relating to x). This represents the rate of change of y with respect to x. Geometrically, $\frac{dy}{dx}$ denotes (Fig. 2) the slope of the curve between variables x and y according to given function at the point (x, y) and we may write



(Fig. 2)

$$\frac{dy}{dx} = \tan\theta$$

Differential Coefficients of Some Simple Functions :

Function $y = f(x)$	Differential Coefficient dy/dx	Function $y = f(x)$	Differential Coefficient dy/dx
1. $y = x^n$	$\frac{dy}{dx} = n x^{n-1}$	5. $y = \sin x$	$\frac{dy}{dx} = \cos x$
2. $y = \text{constant}$	$\frac{dy}{dx} = 0$	6. $y = \cos x$	$\frac{dy}{dx} = -\sin x$
3. $y = \log_e x$	$\frac{dy}{dx} = \frac{1}{x}$	7. $y = \tan x$	$\frac{dy}{dx} = \sec^2 x$
4. $y = \log_a x$	$\frac{dy}{dx} = \frac{1}{x}\log_a e = \frac{1}{x \log_e a}$	8. $y = x$	$\frac{dy}{dx} = 1$

Differential Coefficient of Product of Two Functions :

Let $y = f_1(x) \cdot f_2(x)$, then

$$\frac{dy}{dx} = (\text{first function}) \frac{d}{dx}(\text{second function}) + (\text{second function}) \frac{d}{dx}(\text{first function})$$

$$= f_1(x) \cdot \frac{d}{dx}[f_2(x)] + f_2(x) \cdot \frac{d}{dx}[f_1(x)]$$

* The topic has been mentioned in the syllabus under Unit 2 (Kinematics). Basically it is essentially required to understand the motion.

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For example, when $y = x^2 \sin x$, then

$$\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2)$$

or
$$\frac{dy}{dx} = x^2 \cdot \cos x + \sin x \cdot 2x = x^2 \cos x + 2x \sin x.$$

Differential Coefficient of Quotient of Two Functions :

Let
$$y = \frac{f_1(x)}{f_2(x)} = \frac{\text{Numerator } (N^r)}{\text{Denominator } (D^r)}$$

Then
$$\frac{dy}{dx} = \frac{D^r \cdot \frac{d}{dx}(N^r) - N^r \cdot \frac{d}{dx}(D^r)}{(D^r)^2}$$

For example, when $y = \frac{\sin x}{x^2}$, then

$$\frac{dy}{dx} = \frac{x^2 \cdot \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 \cos x - \sin x\,(2x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3}$$

Applications in Physics : In Physics we study the variation of one variable with respect to other varia In such cases the use of differentiation is very useful. For example,

(i) Rate of change of displacement (s) with respect to time (t) *i.e.*, $\left(\frac{ds}{dt}\right)$ gives the velocity.

(ii) Rate of change of velocity (v) with respect to time (t) *i.e.*, $\frac{dv}{dt}$ gives the acceleration.

(iii) Rate of fall of temperature (θ) with respect to time (t) *i.e.*, $\left(-\frac{d\theta}{dt}\right)$ gives the rate of cooling.

While studying the variation of one variable (say y) with respect to other variable (say x), the natu variation is also determined as follows :

(a) When $\frac{dy}{dx} > 0$ (*i.e.*, positive) then y changes in the same way as is done in x (*i.e.*, if x is incre y also increases and if x is decreased, y also decreases).

(b) When $\frac{dy}{dx} < 0$ (*i.e.*, negative) then y changes in opposite way as done in x (*i.e.*, if x is incre y decreases and if x is decreased, y increases).

(c) When $\frac{dy}{dx} = 0$ then y does not change with x (*i.e.*, y = constant). In other words, y is indepe of x.

In problems of kinematics, differentiation is used to determine the velocity and acceleration as foll

(i) When displacement s (or distance s) is expressed in terms of time t *i.e.*, $s = f(t)$, then veloci speed) is determined by differentiating s with respect to t.*

* Rate of change of displacement (with time) is called velocity.

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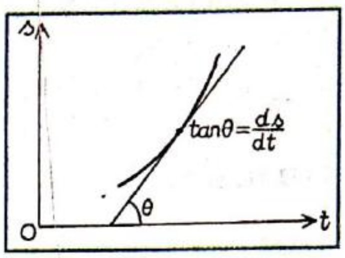
$$v = \frac{ds}{dt}$$

The velocity so obtained is called the instantaneous velocity (*i.e.*, velocity at particular instant t).

When graph is given between s and t (Fig. 3), then velocity at a point is given by the slope of the tangent drawn at that point.

$$v = \tan\theta = \frac{ds}{dt}$$



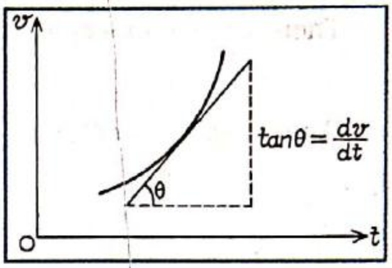
(Fig. 3)

(ii) When velocity v (or speed v) is expressed in terms of time t *i.e.*, $v = f(t)$, then acceleration is determined by differentiating v with respect to t.*

$$a = \frac{dv}{dt}$$

This acceleration is called instantaneous acceleration at time t.

When the graph is given between time (t) and velocity (v) (Fig. 4), then slope of the tangent at a point gives the instantaneous acceleration at that point.



(Fig. 4)

$$a = \tan\theta = \frac{dv}{dt}$$

(iii) In terms of differential coefficient, the acceleration is also given by

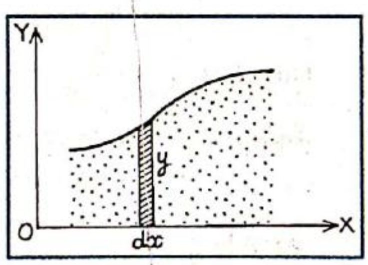
$$a = \frac{dv}{dt} = \frac{ds}{dt} \cdot \frac{dv}{ds}$$

or $$a = v \cdot \frac{dv}{ds} \qquad \left[\because \frac{ds}{dt} = v\right]$$

(B) Integration

Literary, integration means summation. Thus, when a parameter is determined in small intervals and then summed, we say that we have integrated the function representing the parameter.

To limit our scope up to the present standard integration is used to determine the area formed by the curve $y = f(x)$ with X-axis. Area of a very thin strip $= y\,dx = f(x)\,dx$. (Fig. 5)



(Fig. 5)

By integrating, $f(x)\,dx$, the area is determined. Integration is represented by the symbol $\int$ and $\int f(x)\,dx$ means we have to integrate $f(x)$ with respect to x.

As a process, "integration is the reverse process of differentiation". It means that if $g(x)$ is the differential coefficient of function $f(x)$, *i.e.*,

$$\frac{d}{dx}[f(x)] = g(x)$$

then integration of $g(x)$ is $f(x)$, *i.e.*,

$$\int g(x)\,dx = f(x) + C$$

The constant C is called the integration constant.

* Rate of change of velocity (with time) is called acceleration.

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This constant is put due to the reason that functions having same differential coefficient may differ constants. Such integrations are called 'indefinite integrals'.

For example,

$$\frac{d}{dx}(4x^2) = 8x, \frac{d}{dx}(4x^2 + 7) = 8x, \frac{d}{dx}(4x^2 - 3) = 8x.$$

Formulae for integration : The most frequently used and important formula is the 'power formula'

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

i.e., increase the power of variable x by 1 and divide it by the increased power. C is the integration constant. For example,

$$\int x^5\, dx = \frac{x^6}{6} + C,$$

$$\int 3x^{11/2}\, dx = 3\int x^{11/2}\, dx = 3 \cdot \frac{x^{13/2}}{13/2} + C = \frac{6}{13}x^{13/2} + C$$

$$\int \frac{5}{x^7}\, dx = 5\int x^{-7}\, dx = 5 \cdot \frac{x^{-7+1}}{-7+1} + C = -\frac{5}{6}x^{-6} + C.$$

Other useful formulae for integration are :

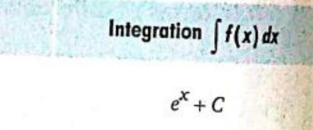
	Function $f(x)$	Integration $\int f(x)\,dx$		Function $f(x)$	Integration $\int f(x)\,dx$
1.	$\int \frac{1}{x} dx$	$\log_e x + C$	4.	$\int e^x\, dx$	$e^x + C$
2.	$\int \sin x\, dx$	$-\cos x + C$	5.	$\int \sec^2 x\, dx$	$\tan x + C$
3.	$\int \cos x\, dx$	$\sin x + C$	6.	$\int dx$	$x + C$

Definite Integrals : When the integration is performed between the given limits (Fig. 6), then it is called definite integral.

$x = x_1$ is called lower limit, $x = x_2$ is called upper limit

Definite integrals are performed in the same way as indefinite integrals (without adding integration constant). Let integral of $f(x)$ is $F(x)$, then

$$\int_{x=x_1}^{x=x_2} f(x)\, dx = [F(x)]_{x_1}^{x_2}$$



(Fig. 6)

Upper limit is first put for variable x (*i.e.*, $x = x_2$), then put minus sign and after it put lower limit for x (*i.e.*, $x = x_1$). Thus

$$\int_{x_1}^{x_2} f(x)\, dx = [F(x)]_{x_1}^{x_2} = F(x_2) - F(x_1).$$

For example,

(i) $\int_1^3 5x^3\, dx = 5\int_1^3 x^3\, dx = 5\left[\frac{x^4}{4}\right]_1^3 = \frac{5}{4}[3^4 - 1^4] = \frac{5}{4}(81 - 1) = \mathbf{100}.$

(ii) $\int_0^{\pi/2} \cos x\, dx = [\sin x]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = \mathbf{1}.$

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Applications in Physics : Integration is used in Physics in two ways :

(i) To determine the area under curves which represents the quantity (having dimensions of the product xy).

(ii) To extend the result on a very small element to the entire range required in the problem.

In kinematics the area under the velocity-time graph represents displacement (= velocity × time) or speed-time graph represents the distance. The area under the acceleration-time graph represents the velocity (= acceleration × time) or speed. Thus

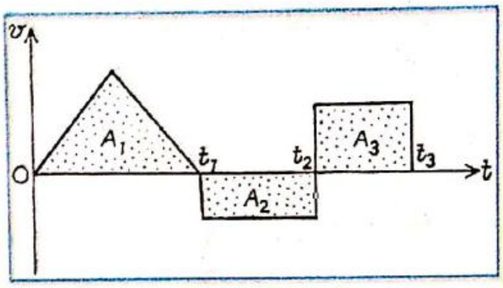
(a) When velocity is expressed as a function of time, then to determine the displacement from time t_1 to t_2, the expression must be integrated between the limits $t = t_1$ to $t = t_2$.

When the v-t graph is given, then the displacement is determined by finding out the area (taking +ve sign for area above time axis and −ve sign for area below time axis) and then adding algebraically (Fig. 7). From the figure, displacement from $t = 0$ to $t = t_3$ is

$$d = A_1 - A_2 + A_3$$

and distance travelled in this time interval is determined by adding the areas taking all with positive sign

$$s = |A_1| + |A_2| + |A_3|$$



(Fig. 7)

(b) When acceleration is expressed as a function of time*, then to determine the velocity, integrate the given expression.

When the a-t graph is given, velocity (or speed) is determined by finding out the area as explained above in determination of displacement.

Velocity = algebraic sum (with sign) of areas

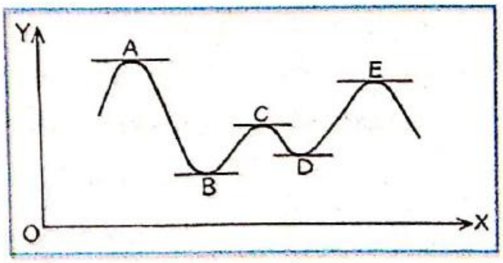
Speed = sum of moduli of areas (taking all areas positive)

2 Concepts of Maximum and Minimum

When a function $y = f(x)$ is to be investigated for maximum and minimum, adopt the following steps :

(i) Find the first derivative i.e., $\frac{dy}{dx}$.

(ii) At points of maximum (A, C, E in figure) and also at points of minimum (B, D in figure), tangent is parallel to X-axis (Fig. 8). For this put



(Fig. 8)

$$\frac{dy}{dx} = 0$$

This gives the values of x for points of maximum and minimum. Let $x = x_1, x_2, x_3, x_4, x_5$.

(iii) To investigate the nature at these points, determine $\frac{d^2y}{dx^2}$ and find its value at above points.

$$\frac{d^2y}{dx^2} = -\text{ve at points of maximum } (A, C, E)$$

To find the maximum values at these points, put $x = x_1, x_3, \ldots$ in $y = f(x)$.

* Since the scope of study is limited to uniformly accelerated motions, acceleration will be given as a constant and a-t graph will be a straight line parallel to time axis.

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Further, $\dfrac{d^2y}{dx^2} = +\text{ve}$ at points of minimum (B, D)

To find the minimum values at these points, put $x = x_2, x_4$ corresponding to these points in $y = f(x)$.

Solved NUMERICAL Problems

Example 1. When $y = 3x^2 - 5x + 7$, find $\dfrac{dy}{dx}$.

Solution. Given $y = 3x^2 - 5x + 7$, then

$$\frac{dy}{dx} = 3 \cdot \frac{d}{dx}(x^2) - 5 \cdot \frac{d}{dx}(x) + 7 \cdot \frac{d}{dx}(x^0)$$
$$= 3 \times 2x - 5 \times 1 + 0 = \mathbf{6x - 5}.$$

Example 2. Given $x = 7t^5 - 4t^2 + 3t + 8$, find $\dfrac{dx}{dt}$ and its value at $t = 2$ s.

Solution. Given $x = 7t^5 - 4t^2 + 3t + 8$

Differentiating x with respect to t

$$\frac{dx}{dt} = 7\frac{d}{dt}(t^5) - 4\frac{d}{dt}(t^2) + 3\frac{d}{dt}(t) + 8\frac{d}{dt}(t^0)$$
$$= 7 \times 5t^4 - 4 \times 2t + 3 \times 1 + 8 \times 0 = \mathbf{35t^4 - 8t + 3}.$$

At $t = 2$ s,

$$\left(\frac{dx}{dt}\right)_{t=2\text{ s}} = 35(2)^4 - 8(2) + 3 = 560 - 16 + 3 = \mathbf{547}.$$

Example 3. The velocity of a moving particle varies with time t as $v = t^3 - 2\sqrt{t} + 1$ where v is in m s^{-1} and t in s. Find the acceleration of the particle at $t = 4$ s.

Solution. Given $v = t^3 - 2\sqrt{t} + 1 = t^3 - 2t^{1/2} + 1$

Differentiating with respect to t,

$$\text{acceleration, } a = \frac{dv}{dt} = 3t^2 - 2 \times \frac{1}{2} \times t^{-1/2} + 0$$
$$= 3t^2 - t^{-1/2} + 0 = 3t^2 - \frac{1}{\sqrt{t}}$$

At time $t = 4$ s,

$$a = 3 \times (4)^2 - \frac{1}{\sqrt{4}} = 48 - \frac{1}{2} = \frac{95}{2} = \mathbf{47.5\ ms^{-2}}.$$

Example 4. The tangent at a point on the velocity time graph of a moving point makes an angle of 135° with time axis. Determine the instantaneous acceleration at that point. Also indicate whether the velocity of the particle is increasing or decreasing at that point?

Solution. Given angle of tangent with abscissa (time axis) $\theta = 135°$

Slope $= \tan\theta = \tan 135° = -1$

Since the slope of v-t graph gives acceleration. Hence, instantaneous acceleration = – 1 unit.

Since the slope $\frac{dv}{dt}$ is negative, the velocity is decreasing with passage of time.

Example 5. Integrate $\int_2^3 x\,dx$.

Solution. $\int_2^3 x\,dx = \left[\frac{x^2}{2}\right]_2^3 = \frac{1}{2}[(3)^2 - (2)^2] = \frac{1}{2}(9-4) = \frac{5}{2}$.

Example 6. The acceleration of a particle is 2 ms^{-2}. The particle starts from rest. What will be its velocity at $t = 5$ s.

Solution. Given $a = 2$ ms^{-2}

$$v = \int_{t=0}^{t=5} a\,dt = 2[t]_0^5 = 2 \times 5 = 10\ \text{ms}^{-1}.$$

Although this can be easily determined by using the equation of motion $v = u + at$.

Example 7. Find the value of $\int_0^x F \cdot dx$, where $F = Kx$.

Solution. $\int_0^x F \cdot dx = \int_0^x Kx\,dx = K\int_0^x x \cdot dx = K\left[\frac{x^2}{2}\right]_0^x = K\left[\frac{x^2}{2} - \frac{0^2}{2}\right] = \frac{1}{2}Kx^2$.

Example 8. Find the value of $\int_{V_1}^{V_2} \frac{1}{V} \cdot dV$.

Solution. $\int_{V_1}^{V_2} \frac{1}{V} \cdot dV = [\log_e V]_{V_1}^{V_2} = [\log_e V_2 - \log_e V_1] = \log_e \frac{V_2}{V_1}$.

Example 9. A spherical balloon is being filled by air so that its volume V is gradually increasing. Find the rate of increase of volume with radius r when $r = 5$ units.

Solution. Volume of spherical balloon $= V = \frac{4}{3}\pi r^3$

$\therefore$ Rate of increase of volume with respect to radius r will be

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi\frac{d}{dr}r^3 = \frac{4}{3}\pi(3r^2) = 4\pi r^2$$

when $r = 5$ units then $\frac{dV}{dr} = 4\pi(5)^2 = 100\pi$.

Example 10. Show that force can be expressed as the product of mass and acceleration.

Solution. $F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = m \cdot a$.

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Chapter 1

Physical World

Syllabus

Scope of Physics and its applications in everyday life. Nature of Physical laws.

1 What is Physics ?

Physics is a branch of science. The word 'Physics' has come from a Greek word 'fusis' meaning 'nature'. The word 'Science' has come from a Latin word 'Scientia' which means 'to know'. Thus, ***Physics is the subject of studying nature and natural phenomena.***

Science : Man has always been curious about the natural phenomena taking place around him, such as, daily repetition of sunrise and sunset, yearly repetition of seasons, regular motion of the moon and other heavenly objects in the sky, etc. He has responded to these phenomena in a systematic manner. The knowledge thus gained which has been transferred from generation to generation, each generation adding new facts to it, is known as 'Science'.

The Scientific Method : The knowledge of science is acquired through a method consisting of the following four steps :

(i) observation of facts.

(ii) proposing theory or hypothesis to explain the observation.

(iii) testing the validity of the theoretical predictions based on the proposed theory.

(iv) modification of the theory, if necessary.

All these steps taken together constitute the scientific method.

It may happen that a new observation or a new measurement shows a discrepancy between an existing theory and the observation. Then the theory is to be modified or even to be replaced by a new theory.

Scope of Physics : The scope of Physics is very wide. We find enormous ranges of length, mass and time in physical processes. For example, the length may range from 10^{-15} m (size of nucleus) to 10^{26} m (size of universe), mass from 10^{-30} kg (mass of electron) to 10^{30} kg (mass of sun) and time-interval from 10^{-22} s (time taken by light to cross a nuclear distance) to 10^{18} s (life of sun). In spite of such a wide range of length, mass and time, the complex physical phenomena involving them can be easily understood. There are three reasons for this :

(i) A quantitative study of various natural phenomena shows that there is some regularity and symmetry even in the most complex phenomenon which helps us in understanding it.

(ii) In spite of the enormous diversity of scales of natural phenomena, all of them can be explained in terms of only a few basic laws. One such law is the law of conservation of energy.

(iii) In any complex phenomenon, we can separate the more important features from the less important ones. We can then understand the complex phenomenon in terms of the more important features and unfold the simplicity hidden behind the complexity.

2 Branches of Physics

For the sake of convenience, different physical phenomena, properties of matter and energy occurring in different forms have been categorised and are studied in different branches of Physics.

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Broadly, the complete study of Physics is carried out in following branches—

(i) Mechanics
(ii) Waves and Oscillations
(iii) Heat and Thermodynamics
(iv) Properties of Matter including Fluid Mechanics
(v) Electrodynamics
(vi) Optics and Relativity
(vii) Atomic Physics
(viii) Nuclear Physics
(ix) Electronics and Communication
(x) Astrophysics
(xi) Computer Science

3 Nature of Physical Laws

Physicists explore the universe. Their investigations range from particles smaller in size than atoms to extremely distant stars. Besides finding facts about nature by observation and experimentation, physicists try to discover the laws that govern these facts.

If any physical phenomenon taking place in nature, several quantities change with time, while some special physical quantities remain constant with time. The later quantities are 'conserved quantities' of nature, and are governed by certain conservation laws. Some of the general conservation laws in nature are the following :

(i) Conservation of Mechanical Energy : For motion under an external conservative* force, the mechanical energy (kinetic energy + potential energy) of a body is a constant. In the familiar example of free-fall of a body under gravity, both, the kinetic energy and the potential energy change continuously with time, but their sum remains constant. If the body is released from rest from some height, the initial potential energy is completely converted into the kinetic energy of the body just before it hits the ground.

(ii) Conservation of Total Energy of an Isolated System : The mechanical energy is conserved if the forces involved are conservative. If some of the forces doing work are non-conservative (such as air friction), part of the mechanical energy is transformed into other forms. In the free-fall example, after the body hits the ground, its initial potential energy is converted into heat, light and sound energies. Although mechanical energy is not conserved, but the *total* energy of the isolated system (falling body plus the surroundings) remains conserved.

(iii) Conservation of Mass Plus Energy : Before Einstein, the law of conservation of mass was considered as a basic conservation law in nature because matter was thought to be indestructible. But, Einstein, by his theory of relativity proved that energy-conservation and mass-conservation are *not* two independent laws; they are unified in a single law. According to it, the ***total (mass + energy) of the universe is conserved.*** If a substance loses an amount Δm of its mass, an equivalent amount ΔE of energy is produced, according to the relation

$$\Delta E = (\Delta m)\, c^2,$$

where c is the speed of light. This is called 'Einstein's mass-energy equivalence relation'.

There may be a decrease in the mass of the universe, but at the same time an equivalent amount of energy is produced. Sun is continuously losing mass, but we are receiving it in the form of energy. One striking example of mass-energy equivalence is the process in which an electron collides with a positron and the two annihilate, giving two photons :

$$e^- + e^+ \longrightarrow \gamma + \gamma.$$

* Conservative force is discussed in chapter 11.

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Here the total mass of the electron and the positron is entirely converted into the energy of the photo[n]

(iv) Conservation of Momentum : Energy is a scalar quantity. All conserved quantities are, however, n[ot] necessarily scalars. The total linear momentum and the total angular momentum (both vectors) of [an] isolated system are also conserved quantities. These laws are derived from Newton's laws of motion [in] mechanics. These are the basic conservation laws of nature in all domains.

(v) Conservation of Charge : Conservation of electric charge of an isolated system is a fundamental la[w] of nature. ***The total amount of charge in an isolated system remains constant. It means charge ca[n] neither be created nor destroyed.*** When a glass rod is rubbed with silk, a positive charge appears o[n] the rod and, at the same time, an equal negative charge appears on the silk. Thus, the net charge on th[e] rod-silk system is zero both, before and after rubbing.

(vi) Conservation of the Number of Nucleons (Mass Number) : In all nuclear reactions, the number o[f] nucleons remains conserved. There is rearrangement of protons and neutrons in different nuclea[r] reactions, but the total number of neutrons and protons is separately equal for the reactants and th[e] products.

In radioactive decay of uranium, the uranium nucleus is converted into thorium nucleus and emits a[n] α-particle :

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}.$$

The total number of nucleons are 238 both, before and after the decay.

Some conservation laws are true for one fundamental force, but not for the other. For example, 'parit[y]' is conserved by the strong electromagnetic interactions but not by the weak interactions. 'Strangenes[s]' is also conserved by the strong force but not by the weak force.

The conservation laws are very useful in practice. They enable us to solve easily the complex problems i[n] mechanics involving different particles and forces. In nuclear and fundamental particle phenomena also using conservation laws of energy and momentum for β-decay, Pauli predicted in 1931 the existence o[f] a new particle (called neutrino) antiparticle of which (*i.e.*, antineutrino) emitted in β-decay along wit[h] the electron.

Conservation Laws and Symmetry : The conservation laws have a strong connection with symmetrie[s] of nature. For example, we observe that laws of nature do not change with time. If we perform a[n] experiment in a laboratory today, and repeat the same experiment after a year under the same condition[s] the results are bound to be the same. This symmetry of nature with respect to time is equivalent to th[e] law of conservation of energy.

The laws of nature are the same everywhere in the universe. However, a phenomenon may differ fro[m] place to place because of differing conditions at different locations. For example, the acceleration d[ue] to gravity at moon is 1/6th that at the earth, but the universal law of gravitation is same both on th[e] moon and the earth. This symmetry of nature with respect to space gives rise to conservation of linea[r] momentum. Similarly, isotropy of space (no preferred direction) gives rise to conservation of angula[r] momentum. Infact, symmetries of time and space play important role in explaining fundamental force[s] in nature.

4 Fundamental Forces in Nature

In our daily life, we come across many types of forces, for example, muscular forces, frictional forces elastic forces, magnetic forces, electric forces, etc. All these forces are included in four basic types o[f] forces operating in nature. These are gravitational forces, weak forces, electromagnetic forces, and strong forces.

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(a) Gravitational Forces : The forces of attraction existing between any two separated bodies due to their masses are called 'gravitational forces'. These forces have following properties:

(i) Gravitational forces between two bodies are always attractive.

(ii) They obey inverse-square law, that is, they vary inversely as the square of the distance between the two bodies.

(iii) They are 'central' forces, that is, they act along the line joining the centres of the two bodies. Hence they are 'conservative' forces.

(iv) Gravitational forces are long-range forces and operate up to very large distances. For example, earth revolves around the sun under gravitational attraction of sun on earth.

(v) These are the weakest forces in nature.

(vi) The field particle of gravitational forces is called 'graviton'. The concept of exchange of a field particle between two bodies explains how the two bodies mutually interact from a distance.

The gravitational forces are responsible for holding the entire earth together, for binding the sun and the planets into the solar system and for binding stars into galaxies.

(b) Weak Forces : The weak forces were discovered during the study of β-emission from nuclei. During β-emission, a neutron inside the nucleus is converted into a proton, an electron and an antineutrino, a particle having no charge and almost zero rest-mass ($n \rightarrow p + e^- + \bar{\nu}$). A continuous variation in the energies of β-particles emitted from a nucleus led to the conclusion that an electron and an antineutrino interact with each other through weak forces. Leptons also interact with each other and with mesons and baryons through weak forces. Thus, ***weak forces are the forces which cause light elementary particles to interact with one another and with heavier particles.*** These are about 10^{26} times stronger than the gravitational forces. The field particle of weak forces is 'neutrino'.

(c) Electromagnetic Forces : These forces include electrostatic force and magnetic force. The force between two stationary charges is 'electrostatic force' while the force between two magnetic poles is 'magnetic force'. These forces are not independent phenomena. A moving charge produces a magnetic field, while a charged particle moving in a magnetic field experiences a force. Thus, electric force and magnetic force are two aspects of electromagnetic force. The electromagnetic forces have following properties :

(i) They may be attractive or repulsive. Like charges and like poles repel each other, while unlike charges and unlike poles attract each other.

(ii) They obey inverse-square law.

(iii) They are 'central' forces and hence they are 'conservative'.

(iv) They operate up to sufficiently large distances.

(v) Electromagnetic forces are about 10^{10} times stronger than the weak forces and about 10^{36} times stronger than the gravitational forces.

(vi) The field particle of electromagnetic forces is 'photon' which carries no charge and has zero rest-mass.

The electromagnetic forces bind the electrons to the nucleus to form atoms, bind atoms together to form molecules, and bind molecules together to form matter.

(d) Strong Forces : These forces are of nuclear origin. They are responsible for holding neutrons and protons together inside atomic nucleus. They have following properties :

(i) The nuclear (strong) forces are basically attractive forces. They exist between neutron and neutron ($n - n$ forces), between proton and proton ($p - p$ forces) and between neutron and proton ($n - p$ forces) inside the nucleus.

(ii) They do not obey inverse-square law.

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(iii) They are 'non-central' and hence 'non-conservative'.

(iv) They are extremely short-range forces and act over distances of the order of 10^{-15} m, that is, within atomic nucleus. At distances appreciably smaller than 10^{-15} m, the nuclear forces become repulsive.

(v) Nuclear forces are the strongest forces found in nature. Because of these forces, even protons inside the nucleus attract one another. The relative magnitudes of gravitational, weak, electromagnetic and strong (nuclear) forces are in the ratios.

$$1 : 10^{26} : 10^{36} : 10^{39}.$$

Thus, nuclear forces are about 10^{39} times stronger than gravitational forces, about 10^{13} times stronger than weak forces and about 10^{3} times stronger than electromagnetic forces.

(vi) The field particle of nuclear forces is π-meson'.

The nuclear forces are found to exist among *heavier* elementary particles. For example, they bind neutrons and protons together to form atomic nuclei of all elements. Mesons and baryons interact with each other through strong forces and hence collectively they are called 'hadrons'.

5 Unification of Forces

We have read that there are four types of fundamental forces in nature. Scientists have always been remain in quest that there must be a single force, of which the four fundamental forces should be the different manifestations. Efforts are being carried out to develop a theory (called united field theory) that may link the properties of the fundamental forces.

The idea of unification is the result of applicability of various laws in different fields. For example, the gravitational force (given by Newton's gravitational law) on one side is responsible for the fall of apples from tree to the ground while on the other side is also responsible for the motion of planets around the sun and also motion of satellites. Similarly electric and magnetic phenomena have been proved inseparable (by Oersted and Faraday experiments), the sound and light waves indicate some similar phenomena etc.

Reductionism : The word is used to device the properties of the whole system by studying the properties of its constituents.

6 Applications of Physics in Technology and Society

In Technology : The technology is the practical application of Physics and other branches of science which has played important role in the development of various industries and in raising the standard of living of the society. Some applications of Physics are as follows :

(i) The Newton's laws of motion find application in the flight of rockets, and Bernoulli's theorem in the designing of aeroplane wings.

(ii) The principles of thermodynamics have been utilised in heat engines (steam, petrol and diesel), refrigerators and air-conditioners.

(iii) The electric bulbs, lamps and tubes are based on conversion of electric energy into light.

(iv) The electromagnetic induction, discovered by Faraday, has found application in electric generators, motors, furnaces, etc.

(v) At hydroelectric power stations generating electricity for homes and industries, the gravitational potential energy of water stored at a height in a dam is converted into electrical energy.

(vi) At thermal power stations, the chemical energy of burning coal is converted into electrical energy.

(vii) The energy released in nuclear fission process is utilised in nuclear reactors which produce electric power.

(viii) The tidal energy in the ocean and the solar energy (based on nuclear fusion process) are being converted into other forms of useful energy.

(ix) The properties and the theory of propagation of electromagnetic waves is applied in radio, television and wireless communication. An X-ray machine is used to identify internal diseases in the body. A geostationary satellite enables us to watch long-distance TV programs, to forecast weather and to make geophysical survey.

(x) Lasers covering an extremely wide field of practical applications make use of the phenomenon of population inversion.

(xi) The calculators and computers are based on digital electronics.

Thus, Physics plays an important role in technology and in our daily lives.

In Society : The development in Physics are having a direct impact on Society. A number of examples can be given :

(i) Telephone, telegraph, telex, teleprinter and now fax are extensively used by housewives, businessmen, industrialists and professionals.

(ii) Radio, television and satellites, as means of communication, have connected all parts of the world together.

(iii) Calculators, computers and lasers have changed the day-to-day life of the mankind.

(iv) Exploration of the new sources of energy has become absolutely necessary for the existence of future generations.

(v) The development of nuclear power has become a threat to the existence of life on the earth.

7 Physics in Relation to Other Sciences

Physics is basis of all the sciences and it has played a key role in their development.

(a) Physics in Relation to Chemistry : The study of atomic structure, X-ray diffraction, radioactivity, etc., has helped chemists to re-arrange elements in the periodic table, to understand the nature of valency and chemical bonding in molecules, and to analyse complex chemical structures.

(b) Physics in Relation to Biological Sciences : The optical microscope of Physics is the main instrument of Biology students who use it to study samples of leaves and animals. The electron microscope is used to study the structure of cells. X-ray and radio-isotopes are used in the detection and treatment of a number of diseases. Electronic instruments are used in recording heart and head ailments.

(c) Physics in Relation to Astronomy : The giant astronomical telescopes of Physics are used to study planets and other heavenly bodies of the solar system. The radio-telescopes have led to the discovery of pulsars and quasars and enabled the astronomers to investigate the structure of the Milky Way.

(d) Physics in Relation to Mathematics : The theories in Physics are based on various mathematical concepts. For example, electromagnetic theory is just a set of mathematical equations. In fact, mathematics has been a powerful tool in the development of classical mechanics, quantum mechanics, electrodynamics and other theoretical topics of Physics.

(e) Physics in Relation to Other Sciences : The laws and procedures used in Physics have led to the development of many new sciences like Biophysics, Geology, Meteorology, Seismology and Oceanography.

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3. Some of the Great Physicists and their Works

S. No.	Scientist	Major Field of Contribution or Discovery	S. No.	Scientist	Major Field of Contribution or Discovery
1.	Archimedes	Principle of buoyancy, lever	14.	C. V. Raman	Inelastic scattering of [illegible] molecules
2.	Galileo	Law of inertia	15.	S. N. Bose	Quantum statistics
3.	Huygens	Wave theory of light	16.	Louis de Broglie	Wave nature of matter
4.	Newton, Isaac	Universal law of gravitation, laws of motion	17.	Heisenberg	Quantum mechanics [illegible] principle
5.	Michael Faraday	Electromagnetic induction	18.	H. R. Hertz	Generation of e.m. waves
6.	J. C. Maxwell	Electromagnetic theory	19.	Niels Bohr	Hydrogen atom
7.	J. C. Bose	Ultra short radio waves	20.	E. O. Lawrence	Cyclotron
8.	Roentgen	X-rays	21.	H. J. Bhabha	Cascade process of cosmic [illegible]
9.	J. J. Thomson	Discovery of electron, atom model	22.	John Bardeen	Transistors. Theory of [illegible] conductivity
10.	M. Curie	Natural radioactivity	23.	C. H. Townes	LASER, MASER
11.	Albert Einstein	Photoelectric effect, theory of relativity, mass-energy relation	24.	James Chadwick	Neutron
12.	Rutherford	Atom model	25.	S. Chandrasekhar	Chandrasekhar limit, structure [illegible] evolution of stars
13.	M. N. Saha	Thermal ionisation	26.	H. Yukawa	Theory of nuclear forces

Solved NUMERICAL Problems

Example 1. Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientists of all time. What do you think did Einstein mean when he said "the most incomprehensible thing about the world is that it is comprehensible"? *(NCERT Problem [illegible])*

Ans. The physical world, when seen by a Layman, presents us with such diversity of things. It seems incomprehensible, *i.e.*, as if it cannot be understood. On study and analysis, the scientists find that physical phenomena from atomic to astronomical ranges can be understood in terms of only a few basic concepts, *i.e.*, the physical world becomes comprehensible. This is what is meant by Einstein's statement made above.

Example 2. Every great physical theory starts as a heresy and ends as a dogma. Give some examples from the history of science of the validity of this incisive remark. *(NCERT Problem [illegible])*

Ans. The statement is true. For example, in ancient times Ptolemy postulated that earth is stationary and all the heavenly bodies like sun, stars, planets etc., revolve around earth. Later, an Italian scientist Galileo was postulated that sun is stationary and earth along with other planets is revolving around sun. Galileo punished by the then authorities for upsetting wrong concept. However, later on [illegible] and Kepler supported Galileo's theory and now it is no more than a dogma.

Example 3. 'Politics is the art of possible'. Similarly 'Science is the art of the soluble'. Explain this beautiful aphorism on the nature and practice of science. *(NCERT Problem [illegible])*

Ans. It is well known that to win over votes, politicians would make anything and everything possible even when they are least sure of the same. The statement that science is the art of the soluble [illegible]

8 Some of the Great Physicists and their Works

S. No.	Scientist	Major Field of Contribution or Discovery	S. No.	Scientist	Major Field of Contribution or Discovery
1.	Archimedes	Principle of buoyancy, lever	14.	C. V. Raman	Inelastic scattering of light by molecules
2.	Galileo	Law of inertia	15.	S. N. Bose	Quantum statistics
3.	Huygens	Wave theory of light	16.	Louis de Broglie	Wave nature of matter
4.	Newton, Isaac	Universal law of gravitation, laws of motion	17.	Heisenberg	Quantum mechanics uncertainty principle
5.	Michael Faraday	Electromagnetic induction	18.	H. R. Hertz	Generation of e.m. waves
6.	J. C. Maxwell	Electromagnetic theory	19.	Niels Bohr	Hydrogen atom
7.	J. C. Bose	Ultra short radio waves	20.	E. O. Lawrence	Cyclotron
8.	Roentgen	X-rays	21.	H. J. Bhabha	Cascade process of cosmic radiation
9.	J. J. Thomson	Discovery of electron, atom model	22.	John Bardeen	Transistors. Theory of super conductivity
10.	M. Curie	Natural radioactivity	23.	C. H. Townes	LASER, MASER
11.	Albert Einstein	Photoelectric effect, theory of relativity, mass-energy relation	24.	James Chadwick	Neutron
12.	Rutherford	Atom model	25.	S. Chandrasekhar	Chandrasekhar limit structure and evolution of stars
13.	M. N. Saha	Thermal ionisation	26.	H. Yukawa	Theory of nuclear forces

Solved NUMERICAL Problems

Example 1. Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientists of all time. What do you think did Einstein mean when he said: "The most incomprehensible thing about the world is that it is comprehensible" ? *(NCERT Problem 1.1)*

Ans. The physical world, when seen by a Layman, presents us with such diversity of things. It seems incomprehensible, *i.e.*, as if it cannot be understood. On study and analysis, the scientists find that the physical phenomena from atomic to astronomical ranges can be understood in terms of only a few basic concepts, *i.e.*, the physical world becomes comprehensible. This is what is meant by Einstein's statement made above.

Example 2. Every great physical theory starts as a hearsay and ends as a dogma. Give some examples from the history of science of the validity of this incisive remark. *(NCERT Problem 1.2)*

Ans. The statement is true. For example, in ancient times Ptolemy postulated that earth is stationary and all the heavenly bodies like sun, stars, planets etc., revolve around earth. Later, an Italian scientist Galileo was postulated that sun is stationary and earth along with other planets is revolving around the sun. Galileo punished by the then authorities for spreading wrong concepts. However, later on Newton and Kepler supported Galileo's theory and now it is no more than a dogma.

Example 3. 'Politics is the art of possible'. Similarly 'Science is the art of the soluble'. Explain this beautiful aphorism on the nature and practice of science. *(NCERT Problem 1.3)*

Ans. It is well known that to win over votes, politicians would make anything and everything possible even when they are least sure of the same. The statement that science is the art of the soluble implies

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that a wide variety of physical phenomena are understood in terms of only a few basic concepts, *i.e.*, there appears to be unity in diversity as if widely different phenomena are soluble and can be explained in terms of only a few fundamental laws.

Example 4. Though Indian has a large base in science and technology which is fast expanding, it is still a long way from realizing its political of becoming a world leader in science. Name some important factors, which in your view have hindered the advancement of science in India. ***(NCERT Problem 1.4)***

Ans. In my view some important factors which have hindered the advancement of science in India are :

(i) Lack of education,

(ii) Poverty, which leads to lack of resources and lack of infrastructure,

(iii) Pressure of increasing population,

(iv) Lack of scientific planning,

(v) Lack of development of work culture and self discipline.

Example 5. No physicist has ever "seen " an electron. Yet, all physicists believe in the existence of electron. An intelligent but superstitious man advances this analogy to argue that 'ghosts' exist even though no one has 'seen' one. How will you refute his argument ? ***(NCERT Problem 1.5)***

Ans. No physicist has ever 'seen' an electron. This is true. But there is so much of evidence that establishes the existence of electrons. On the contrary, there is hardly any evidence, direct or indirect to establish the existence of ghosts.

Example 6. The industrial revolution in England and Western Europe more than two centuries ago was triggered by some key scientific and technological advances. What are these advances? ***(NCERT Problem 1.7)***

Ans. Some of the key advances responsible for industrial revolution were as given below :

1. Invention of steam engine by James Watt.
2. Invention of flying shuttle by John Key and Power Loom by cartright brought revolution in textile industry.
3. Invention of safety lamp by Humphry Davy helped workers to work safely in mines.
4. Setting up of blast furnace helped to convert low grade iron into steel.

Example 7. Science, like any knowledge, can be put to good or bad use, depending on the user. Given below are some of the applications of science. Formulate your views on whether the particular application is good, bad or something that cannot be so clearly categorized.

(a) Mass vaccination against small pox to curb and finally eradicate this disease from the population. (This has already been successfully done in India.)

(b) Television for eradication of illiteracy and for mass communication for news and ideas.

(c) Prenatal sex determination.

(d) Computers for increase in work efficiency.

(e) Putting artificial satellites around the Earth.

(f) Development of nuclear weapons.

(g) Development of new and powerful techniques of chemical and biological warfare.

(h) Purification of water for drinking.

(i) Plastic surgery.

(j) Cloning. ***(NCERT Problem 1.11)***

Ans. (a) Mass vaccination is good as it is used to protect the society from the dreaded diseases like small pox.

(b) Good to educate, entertain and create awareness amongst people.

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