CONTENTS

1 KNOWING OUR NUMBERS

INTRODUCTION

Many thousand years ago, people knew only small numbers. Gradually, they learnt how to handle large numbers and express large numbers in symbols. All this came through collective efforts of human beings. Their path was not easy, they struggled all along the way. In fact, the development of whole Mathematics can be understood this way. As human beings progressed, there was greater need for development of Mathematics and as a result Mathematics grew further and faster.

We enjoyed working with numbers in our previous classes. We have added, subtracted, multiplied and divided them. We also looked for patterns in numbers and have done many other interesting things with numbers. In this chapter, we shall move forward on such interesting things with a bit of review and will learn:

- Natural numbers
- Place value and face value of a digit in a number
- Indian system of numeration
- International system of numeration
- Uses of commas in reading and writing large numbers
- Comparison of numbers
- Formation of numbers
- Conversion of units of length, mass and capacity
- Word problems on large numbers
- Estimation of numbers to a certain degree of accuracy

NATURAL NUMBERS

In earlier classes, we have been dealing with counting numbers 1, 2, 3, 4, ... etc. These counting numbers are called **natural numbers.** The **smallest** natural number is 1.

Natural numbers are used in many different contexts and in many ways. Natural numbers help us in counting concrete objects. We can count objects in large numbers. For example, the number of students of your school, the number of people of your city (town or village). As the process of counting is endless, there is no **largest** natural number.

Digits or figures

Any number (howsoever large) can be written with the use of ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each of these symbols is called a **digit** or a **figure.**

A number system involves counting in **tens**. When we speak of counting in tens, it simply means that we are thinking of collections by tens. Ten is called the **base** of the system.

In our number system:

Ten 'units or ones' make one ten *i.e.* $10 \times 1 = 10$

Ten 'tens' make one hundred *i.e.* $10 \times 10 = 100$

Ten 'hundreds' make one thousand *i.e.* $10 \times 100 = 1000$ and so on.

Place value and face value of a digit in a number

Let us consider the number 7302.

As the digit 7 occupies thousand's place, the *place* (*or local*) *value* of the digit $7 = 7 \times 1000 = 7000$.

Similarly, the place value of the digit $3 = 3 \times 100 = 300$,

the place value of the digit $0 = 0 \times 10 = 0$ and

the place value of the digit $2 = 2 \times 1 = 2$.

Thus, the ***place*** *(or* ***local****)* ***value*** *of a (non-zero) digit in a number depends upon the place it occupies in the given number; and the place value of the digit 0 is always 0 regardless of the place it occupies in the given number.*

Let us consider the number 5354.

The place value of the digit 5 at ten's place $= 5 \times 10 = 50$ and the place value of the digit 5 at thousand's place $= 5 \times 1000 = 5000$ but the face (true or intrinsic) value of both the fives is 5.

Thus, the ***face*** *(****true*** *or* ***intrinsic****)* ***value*** *of a digit in a number is the digit itself, regardless of the place it occupies in the number.*

Remark

To find the place value of a digit in a number, multiply the digit by the value of the place it occupies.

Numerals and numeration

A number can be written in digits as well as in words.

A single digit or a group of digits representing a number is called a **numeral**. For example: 6, 28, 304 and 7659 are numerals.

Thus, a numeral is a symbolic **representation** of a number. Hereafter, we shall use the words number and numeral in the same sense.

Writing a number in words is called **numeration**.

To read and write numbers, two systems of numeration in common use are:

(*i*) Indian system (*ii*) International system.

INDIAN SYSTEM OF NUMERATION

In the Indian system:

100×1000 *i.e.* 100000 is called **lakh**,

100×100000 *i.e.* 10000000 is called **crore** and so on.

In the Indian place value chart:

- The places ones (or units), tens and hundreds together are called **ones (or units) period.**
- The places thousands and ten thousands together are called **thousands period.**
- The places lakhs and ten lakhs together are called **lakhs period.**
- The places crores and ten crores together are called **crores period** and so on.

The Indian place value chart is shown below:

Periods	...	*Crores*		*Lakhs*		*Thousands*		*Ones*		
Places		Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
		TC	C	TL	L	TTh	Th	H	T	O
Numbers		100000000	10000000	1000000	100000	10000	1000	100	10	1

Writing numbers

To write numbers (numerals) for number names, consider the following examples:

(*i*) In a number, if a place is vacant then put 0 at that place. The number 'seven thousand thirty two' consists of a collection of 7 thousands, 3 tens and 2 units *i.e.* it consists of 7 thousands, 0 hundreds, 3 tens and 2 units.

This number is represented by writing 7 at thousand's place, 0 at hundred's place, 3 at ten's place and 2 at one's place.

Thus, the number is written as '7032'.

The **expanded form** of the number is

$$7032 = 7 \times 1000 + 0 \times 100 + 3 \times 10 + 2$$
$$= 7000 + 0 + 30 + 2$$
$$= 7000 + 30 + 2$$

(*ii*) The number 'three crore seventy lakh twenty five thousand two' consists of 3 crores, 70 lakhs, 25 thousands and 2 ones *i.e.* it consists of 3 crores, 7 ten lakhs, 0 lakhs, 2 ten thousands, 5 thousands, 0 hundreds, 0 tens and 2 ones.

This number is represented by writing 3 at crore's place, 7 at ten lakh's place, 0 at lakh's place, 2 at ten thousand's place, 5 at thousand's place, 0 at hundred's place, 0 at ten's place and 2 at one's place.

Thus, the number is written as '37025002'.

The above numbers are written in the place value chart as under:

Periods	...	*Crores*		*Lakhs*		*Thousands*		*Ones*		
Places		TC	C	TL	L	TTh	Th	H	T	O
Numbers							7	0	3	2
			3	7	0	2	5	0	0	2

Remarks

* If a place is vacant in a number, put 0 at that place.
* Never write 0 at the higher place of a number *i.e.* do not put 0 as the extreme left digit of a number. For example, the number seven hundred thirty five is written as 735 and not as 0735.

Reading numbers

To read large numbers, we insert commas after each period starting from ones period. In the Indian system, the various periods from the right are: ones, thousands, lakhs, crores and so on.

For example, the number 29307546 in the Indian system can be written as

2, 93, 07, 546.

It is read as "two crore ninety three lakh seven thousand five hundred forty six".

INTERNATIONAL SYSTEM OF NUMERATION

In the International system:

1000×1000 *i.e.* 1000000 is called **million**,

1000×1000000 *i.e.* 1000000000 is called **billion** and so on.

In the International place value chart:

- The places ones (or units), tens and hundreds together are called **ones period.**
- The places thousands, ten thousands and hundred thousands together are called **thousands period.**
- The place millions, ten millions and hundred millions together are called **millions period** and so on.

The International place value chart is shown below:

Periods	...	*Millions*			*Thousands*			*Ones*		
Places		Hundred millions HM	Ten millions TM	Millions M	Hundred thousands HTh	Ten thousands TTh	Thousands Th	Hundreds H	Tens T	Ones O
Numbers		100000000	10000000	1000000	100000	10000	1000	100	10	1

Reading and writing numbers

To read large numbers, we insert commas after each period starting from the ones period. In the International system, the various periods from the right are:

ones, thousands, millions, billions and so on. For example, the number 3 044 705 693 in the International system can be written as

3, 044, 705, 693.

It is read as "three billion forty four million seven hundred five thousand six hundred ninety three".

To write numbers for number names, consider the following example:

The number 'twenty three million seven hundred thirty thousand five hundred eight' consists of a collection of 2 ten millions, 3 millions, 7 hundred thousands, 3 ten thousands, 5 hundreds and eight ones *i.e.* it consists of 2 ten millions, 3 millions, 7 hundred thousands, 3 ten thousands, 0 thousands, 5 hundreds, 0 tens and 8 ones.

This number can be represented by writing 2 at ten million's place, 3 at million's place, 7 at hundred thousand's place, 3 at ten thousand's place, 0 at thousand's place, 5 at hundred's place, 0 at ten's place and 8 at one's place.

Thus, the number is written as 23, 730, 508.

This number in the International place value chart is written as under:

Periods	...	*Millions*			*Thousands*			*Ones*		
Places		HM	TM	M	HTh	TTh	Th	H	T	O
23, 730, 508			2	3	7	3	0	5	0	8

Uses of commas

- Use of commas is very helpful in reading and writing large numbers.
- Commas are used in separating periods.
- In the Indian system of numeration, the first comma comes after 3 digits from the right and the next comma comes after every 2 digits.
- In the International system of numeration, the commas come after every 3 digits from the right.
- While writing number names, we do not use commas.

Remark

By comparing the Indian and the International place value charts, we find that:

1 million = 10 lakhs, 10 millions = 1 crore,

100 millions = 10 crores and 1 billion = 100 crores

Example 1. *What is the place value of the digit 3 in the number 2370186?*

Solution. The given number (in Indian system) can be written as 23,70,186.

As the digit 3 occupies lakh's place, so its place value

$= 3 \times 100000 = 300000.$

Example 2. *Write the face value and the place value of each of the underlined digit in the numeral*

<u>6</u> <u>0</u> 3 <u>8</u> <u>1</u> 2 <u>4</u>

Solution. Using Indian system and inserting comma after each period, the given numeral can be written as

<u>6</u> <u>0</u>, 3<u>8</u>, <u>1</u>2<u>4</u>.

The underlined digits in the given numeral are 6, 0, 8, 1 and 4.

Digit	Face value	Place value
6	6	6 × 1000000 = 6000000
0	0	0 × 100000 = 0
8	8	8 × 1000 = 8000
1	1	1 × 100 = 100
4	4	4 × 1 = 4

Example 3. *Determine the product of place values of two 6's in 306165.*

Solution. The given number is 306165.

The place value of the digit 6 in thousand's place = 6 × 1000 = 6000,

the place value of the digit 6 in ten's place = 6 × 10 = 60.

∴ The product of the place values of two 6's in the given number

= 6000 × 60 = 360000.

Example 4. *Using International system of numbers, find the difference of the place values of two 5's in 6857021530.*

Solution. The given number in International system can be written as

6, 857, 021, 530.

The place value of 5 at hundred's place = 5 × 100 = 500.

The place value of 5 at ten million's place

= 5 × 10,000,000 = 50,000,000.

∴ The required difference = 50,000,000 – 500

= 49, 999, 500.

Exercise 1.1

1. Write the smallest natural number. Can you write the largest natural number?
2. Fill in the blanks:
   (*i*) 1 lakh = ... ten thousand
   (*ii*) 1 million = ... hundred thousand
   (*iii*) 1 crore = ... ten lakh
   (*iv*) 1 billion = ... hundred million.
3. Insert commas suitably and write each of the following numbers in words in the Indian system and the International system of numeration:
   (*i*) 506723 (*ii*) 180018018
4. Write the following numbers in expanded form:
   (*i*) 750687 (*ii*) 5032109
5. Write the following numbers in figures:
   (*i*) Seven lakh three thousand four hundred twenty
   (*ii*) Eighty crore twenty three thousand ninety three
   Also write the above numbers in the place value chart.

6. Write each of the following numbers in numeral form and place commas correctly:
   (*i*) Seventy three lakh seventy thousand four hundred seven.
   (*ii*) Nine crore five lakh forty one.
   (*iii*) Fifty eight million four hundred twenty three thousand two hundred two.
7. Write the face value and place value of the digit 6 in the number 756032.
8. Find the difference between the place value and the face value of the digit 9 in the number 229301.
9. Determine the difference of the place value of two 7's in 37014472 and write it in words in International system.
10. Determine the product of place value and the face value of the digit 4 in the number 5437.
11. Find the difference between the number 895 and that obtained on reversing its digits.

COMPARISON OF NUMBERS

To compare two natural numbers, we adopt the following procedure:

Step 1. If the number of digits in the given numbers is unequal, then the number having more digits is greater.

Step 2. If the number of digits in the given numbers is equal, then compare the digits at the highest place, the number having greater digit (at the highest place) is greater. If the digits at the highest place are equal, then compare the digits at the next highest place, the number having greater digit (at the next highest place) will be greater, and so on.

For example:

(*i*) Consider the numbers 53757 and 400381.
The number of digits in 53757 = 5,
the number of digits in 400381 = 6.
∴ 400381 > 53757.

(*ii*) Consider the numbers 46301 and 49012.
The number of digits in 46301 = 5,
the number of digits in 49012 = 5.
∴ Both the numbers have equal digits.
The digit in the highest place in 46301 = 4,
the digit in the highest place in 49012 = 4.
∴ Both numbers have same digit at the highest place.
The digit in the second highest place in 46301 = 6,
the digit in the second highest place in 49012 = 9.
As 9 > 6, therefore, 49012 > 46301.

4 ⑥ 3 0 1
↕ ↓
4 ⑨ 0 1 2

Example 1. *Arrange the following numbers in ascending order:*
4378, 64805, 64389.

Solution. The number of digits in 4378 = 4,
the number of digits in 64805 = 5 and
the number of digits in 64389 = 5.

> Ascending means smaller to greater

To compare the numbers 64805 and 64389; write these numbers as shown:

6	4	(8)	0	5
6	4	(3)	8	9

Since 8 > 3, 64805 > 64389

∴ The given numbers in ascending order are 4378, 64389, 64805.

Example 2. *Arrange the following numbers in descending order:*
25047, 374504, 374318, 25407, 3700911

Solution. The number of digits in 25047 = 5,
the number of digits in 374504 = 6,
the number of digits in 374318 = 6,
the number of digits in 25407 = 5 and
the number of digits in 3700911 = 7.

> Descending means greater to smaller

To compare 25047 and 25407, write the numbers as shown:

2	5	(0)	4	7
2	5	(4)	0	7

Since 4 > 0, 25407 > 25047.

To compare 374504 and 374318, write the numbers as shown:

3	7	4	(5)	0	4
3	7	4	(3)	1	8

Since 5 > 3, 374504 > 374318.

Thus, the given numbers in descending order are:
3700911, 374504, 374318, 25407, 25047.

FORMATION OF NUMBERS

We can form numbers from the given digits with or without repetition of digits.

Example 3. *Write all possible 2-digit numbers that can be formed by using the digits 2, 7 and 9 when repetition of digits is not allowed.*

Solution. We are required to write 2-digit numbers. The given digits are 2, 7, 9 and the repetition of digits is not allowed.

Out of the given digits, the possible ways of choosing two digits are

2, 7; 2, 9; 7, 9

Using the digits 2 and 7, the numbers are 27 and 72.
Similarly, *using the digits 2 and 9*, the numbers are 29 and 92.
Using the digits 7 and 9, the numbers are 79 and 97.
Hence, all possible 2-digit numbers are

27, 72, 29, 92, 79, 97.

Tens	*Ones*
2	7
7	2
2	9
9	2
7	9
9	7

Example 4. *Write all possible 3-digit numbers that can be formed by using the digits 1, 9 and 4 when repetition of digits is not allowed.*

Solution. We are required to write 3-digit numbers using the digits 1, 9, 4 and the repetition of digits is not allowed.

Keeping 1 at unit's place, 3-digit numbers are 941 and 491.

Keeping 4 at unit's place, 3-digit numbers are 914 and 194.

Keeping 9 at unit's place, 3-digit numbers are 419 and 149.

Hence, all possible 3-digit numbers are 941, 491, 914, 194, 419 and 149.

Hundreds	*Tens*	*Ones*
9	4	1
4	9	1
9	1	4
1	9	4
4	1	9
1	4	9

Example 5. *Find the number of 3-digit numbers that can be formed by the digits 7, 3 and 0 using each digit only once.*

Solution. The given digits are 7, 3, 0 and the repetition of digits in any number is not allowed.

Keeping 0 at unit's place, 3-digit numbers are 730 and 370.

Keeping 0 at ten's place, 3-digit numbers are 703 and 307.

Hundreds	*Tens*	*Ones*
7	3	0
3	7	0
7	0	3
3	0	7

The digit 0 cannot be put at hundred's place because that would make the number only 2-digited.

$\therefore$ The 3-digit required numbers are 730, 370, 703 and 307. Hence, the total number of 3-digit required numbers = 4.

Example 6. *Write all possible 2-digit numbers that can be formed by using the digits 2, 7 and 8; repetition of digits is allowed.*

Solution. We are required to write 2-digit numbers. The given digits are 2, 7 and 8 and repetition of digits is allowed.

From the given digits, the possible ways of choosing two digits are 2, 7; 2, 8; 7, 8.

Using digits 2 and 7, the numbers are 27, 72, 22 and 77.

Using digits 2 and 8, the numbers are 28, 82, 22 and 88.

But the number 22 has already been taken, so the new numbers are 28, 82, 88.

Using the digits 7, 8, the numbers are 78, 87, 77 and 88.

But the numbers 77 and 88 have already been taken, so the new numbers are 78 and 87.

Hence, the required numbers are 27, 72, 22, 77, 28, 82, 88, 78 and 87.

Smallest and greatest natural numbers

Smallest 1-digit natural number = 1 and

greatest 1-digit natural number = 9 ;

smallest 2-digit natural number = 10 and

greatest 2-digit natural number = 99 ;

smallest 3-digit natural number = 100 and

greatest 3-digit natural number = 999, and so on.

Look at the pattern:

$$9 + 1 = 10,$$
$$99 + 1 = 100,$$
$$999 + 1 = 1000,$$
$$9999 + 1 = 10000, \text{ and so on.}$$

Thus, we have:

greatest 1-digit number + 1 = smallest 2-digit number,
greatest 2-digit number + 1 = smallest 3-digit number,
greatest 3-digit number + 1 = smallest 4-digit number,
greatest 4-digit number + 1 = smallest 5-digit number, and so on.

Example 7. *How many 5-digit numbers are there in all?*

Solution. The greatest 5-digit number = 99999

The greatest 4-digit number = 9999

∴ The total number of 5-digit numbers = 99999 – 9999 = 90000.

Example 8. *Write the greatest number and the smallest number of 4 digits that can be formed by the digits 3, 7, 8 and 1; using each digit only once.*

Solution. The given digits are 3, 7, 8, 1 and the repetition of digits is not allowed. For the *greatest 4-digit number*, the digit in the thousand's place has to be the greatest, which is 8; followed by the next greatest digit 7 in the hundred's place and so on.

Thousands	*Hundreds*	*Tens*	*Ones*
8	7	3	1

∴ The greatest 4-digit number that can be formed = 8731.

For the *smallest 4-digit number*, we have to follow just the reverse. The digit in the thousand's place has to be the smallest, which is 1; followed by the next smallest digit 3 at the hundred's place and so on.

Thousands	*Hundreds*	*Tens*	*Ones*
1	3	7	8

∴ The smallest 4-digit number that can be formed = 1378.

Example 9. *Write the smallest 4-digit number using each of the digits 3, 7, 0 and 9 only once.*

Solution. The given digits are 3, 7, 0, 9 and repetition of digits is not allowed. Out of the given digits, 0 is the smallest, but we cannot put 0 at the thousand's place because that would make the number only 3-digited.

Thousands	*Hundreds*	*Tens*	*Ones*
3	0	7	9

Thus, the thousand's place has to be filled by the smallest non-zero digit. Obviously, it is 3.

The hundred's place has to be filled by the smallest digit out of the remaining digits 7, 0, 9, which is 0; followed by the next smallest digit 7 at ten's place and so on.

∴ The smallest 4-digit number that can be formed = 3079.

Example 10. *Write the greatest 5-digit number having three different digits.*

Solution. The greatest three different digits are 9, 8, 7.

The greatest 5-digit number having three different digits = 9 | 9 | 9 | 8 | 7

Example 11. *Write the smallest 6-digit number having four different digits.*

Solution. The smallest four different digits are 0, 1, 2, 3.

The smallest 6-digit number having four different digits = 1 | 0 | 0 | 0 | 2 | 3

Example 12. *Write the greatest and the smallest 4-digit numbers using four different digits with the condition that 5 occurs at ten's place.*

Solution. Greatest number = 9 | 8 | 5 | 7

Smallest number = 1 | 0 | 5 | 2

Example 13. *Write the largest 4-digit number, using any one digit twice, formed from the digits 4, 8, 1 and 7.*

Solution. The given digits are 4, 8, 1 and 7.

Since one digit is to be used twice, to get largest number, we have to use the digit 8 twice. The other two bigger digits from the given digits are 7 and 4.

The largest 4-digit number, using one digit twice, formed from the given digits = 8874.

Example 14. *Write the smallest 5-digit number, using any one digit twice, formed from the digits 3, 2, 0, 7, 6, 5 and 9.*

Solution. The given digits are 3, 2, 0, 7, 6, 5 and 9.

Since one digit is to be used twice, to get smallest number, we have to use the digit 0 twice. The other three smaller digits from the given digits are 2, 3 and 5.

We know that 0 cannot be put at the highest place because that would make the number only 4-digited.

The smallest 5-digit number, using one digit twice, formed from the given digits = 20035.

Example 15. *Keeping the place value of digit 8 in the number 3680591 same, rearrange the digits of the number to get the greatest number and the smallest number of 7 digits.*

Solution. Keeping the place value of digit 8 in the number 3680591 and rearranging the digits of the given number, we get :

greatest number of 7 digits = 9685310;

smallest number of 7 digits = 1083569.

Exercise 1.2

1. Use the appropriate symbol < or > to fill in the blanks:

   (*i*) 173 ... 189 (*ii*) 1058 ... 1074 (*iii*) 8315 ... 8037.

2. In each of the following pairs of numbers, state which number is smaller:

   (*i*) 553, 503 (*ii*) 41338, 1139 (*iii*) 25431, 24531.

3. Find the greatest and the smallest numbers in each row:
   (*i*) 71834, 75284, 571, 2333, 594 (*ii*) 9853, 7691, 9999, 12002.
4. Arrange the following numbers in ascending order:
   304, 340, 34, 43, 430.
5. Arrange the following numbers in descending order:
   53, 7333, 553, 7529, 335.
6. Write all possible 2-digit numbers that can be formed by using the digits 2, 3 and 4. Repetition of digits is not allowed. Also find their sum.
7. Write all possible 3-digit numbers using the digits 3, 1 and 5. Repetition of digits is not allowed.
8. Write all possible 3-digit numbers using the digits 7, 0 and 6. Repetition of digits is not allowed. Also find their sum.
9. Write all possible 2-digit numbers using the digits 4, 0 and 9. Repetition of digits is not allowed. Also find their sum.
10. Write all possible 2-digit numbers that can be formed by using the digits 3, 7 and 9. Repetition of digits is allowed.
11. Write all possible numbers using the digits 3, 1 and 5. Repetition of digits is not allowed.
    [**Hint.** Here the numbers can be of one, two or three digits.]
12. How many 6-digit numbers are there in all?
13. Write down the greatest number and the smallest number of 4 digits that can be formed by the digits 7, 5, 0 and 4 using each digit only once.
14. Rearrange the digits of the number 5701024 to get the largest number and the smallest number of 7 digits.
15. Keeping the place value of digit 3 in the number 730265 same, rearrange the digits of the given number to get the largest number and smallest number of 6 digits.
16. Form the smallest and greatest 4-digit numbers by using any one digit twice from the digits:
    (*i*) 5, 2, 3, 9 (*ii*) 6, 0, 1, 4 (*iii*) 4, 6, 1, 5, 8.
17. Write (*i*) the greatest number of 6 digits (*ii*) the smallest number of 7 digits. Also find their difference.
18. Write the greatest 4-digit number having distinct digits.
19. Write the smallest 4-digit number having distinct digits.
20. Write the greatest 6-digit number using three different digits.
21. Write the smallest 7-digit number using four different digits.
22. Write the greatest and the smallest 4-digit numbers using four different digits with the conditions as given:
    (*i*) Digit 7 is always at units place.
    (*ii*) Digit 4 is always at tens place.
    (*iii*) Digit 9 is always at hundreds place.
    (*iv*) Digit 2 is always at thousands place.

CONVERSION OF UNITS

Units of money

	In symbols
100 paise = 1 rupee	100 p = ₹ 1

Units of time

60 seconds = 1 minute	60 s = 1 min
60 minutes = 1 hour	60 min = 1 h
24 hours = 1 day	24 h = 1 day
7 days = 1 week	
30 days = 1 month	
12 months = 1 year	
365 days = 1 year	(366 days = 1 leap year)

Units of length

Different units of measuring length are kilometre (km), metre (m), centimetre (cm) and millimetre (mm).

These units of measuring length are connected by the relations:

1 kilometre = 1000 metre

1 metre = 100 centimetre

1 centimetre = 10 millimetre

Note that:

1 m = 100 cm = 100 × 10 mm = 1000 mm and

1 km = 1000 m = 1000 × 1000 mm = 1000000 mm.

> **Remember**
> - 1 km = 1000 m
> - 1 m = 100 cm
> - 1 cm = 10 mm

Units of mass

Different units of measuring mass are kilogram (kg), gram (g) and milligram (mg).

These units of measuring mass are connected by the relations:

1 kilogram = 1000 gram

1 gram = 1000 milligram

Note that:

1 kg = 1000 g = 1000 × 1000 mg = 1000000 mg.

> **Remember**
> - 1 kg = 1000 g
> - 1 g = 1000 mg

Units of capacity

Different units of measuring capacity are kilolitre (kL), litre (L) and millilitre (mL).

These units of measuring capacity are connected by the relations:

1 kilolitre = 1000 litre

1 litre = 1000 millilitre

Note that:

1 kL = 1000 L = 1000 × 1000 mL = 1000000 mL.

> **Remember**
> - 1 kL = 1000 L
> - 1 L = 1000 mL

In the above units of measuring length, mass and capacity, we have used some common words like kilo, milli and centi. Remember that among these kilo is the largest and milli is the smallest.

> kilo means 1000 times greater
> milli means 1000 times smaller
> centi means 100 times smaller

WORD PROBLEMS ON LARGE NUMBERS

We illustrate the operations on large numbers with the help of following examples:

Example 1. *On 1st January 2016, the population of three cities of India was 28,70,577; 74,06,509 and 63,88,003. What was the total population of the three cities?*

Solution. The total population of the three cities is the sum of the number of people in the three cities.

```
     28,70,577
     74,06,509
  +  63,88,003
  ------------
   1,66,65,089
```

Hence, the total population of the three cities was 1,66,65,089.

Example 2. *Rajan Book Store sold books worth ₹2,85,891 in the first week of June and worth ₹4,00,768 in the second week of the same month. How much was the sale for the two weeks together? In which week was the sale greater and by how much?*

Solution. Sale in the first week = ₹2,85,891

and sale in the second week = ₹4,00,768

Total sale for the two weeks = ₹2,85,891 + ₹4,00,768

Hence, the total sale for the two weeks = ₹6,86,659.

Both the numbers 2,85,891 and 4,00,768 are 6-digited,

since 4 > 2, therefore 4,00,768 > 2,85,891.

Hence, the sale of the second week was greater than that of first week.

The sale of the second week was greater than that of first week by ₹1,14,877.

```
   2,85,891
 + 4,00,768
 ----------
   6,86,659

   4,00,768
 - 2,85,891
 ----------
   1,14,877
```

Example 3. *The number of sheets of paper available for making notebooks is 78,000. Each sheet of paper makes 8 pages of a notebook. If each notebook contains 192 pages, find the number of notebooks that can be made from the paper available.*

Solution. Number of sheets of paper available = 78,000

As each sheet of paper makes 8 pages of a notebook, number of pages

= 78,000 × 8

∴ The number of pages available for making notebooks

= 6,24,000.

Since 192 pages make 1 notebook, therefore, 6,24,000 pages make 6,24,000 ÷ 192 = 3250 notebooks.

Hence, the number of notebooks that can be made

= 3250.

```
    78,000
       × 8
  --------
  6,24,000

        3250
  192 ) 624000
       - 576
       -----
          480
        - 384
        -----
          960
        - 960
        -----
            0
```

Example 4. *The distance between Ritu's house and her school is 1 km 875 m. Everyday she walks both ways. Find the total distance covered by her in 6 days.*

Solution. Distance covered by Ritu in walking one way = 1 km 875 m

= (1 × 1000 + 875) m

= 1875 m

Convert the data in m

∴ Distance covered by Ritu in walking both ways in a day

= (2 × 1875) m

= 3750 m

$$\begin{array}{r} 1875 \\ \times\ 2 \\ \hline 3750 \end{array}$$

∴ Total distance covered in 6 days

= (6 × 3750) m = 22500 m

= (22 × 1000) m + 500 m = 22 km 500 m

$$\begin{array}{r} 3750 \\ \times\ 6 \\ \hline 22500 \end{array}$$

Example 5. *To stitch a shirt, 2 m 15 cm cloth is needed. Out of 40 m cloth, how many shirts can be stitched and how much cloth will remain?*

Solution. The length of cloth available = 40 m

= (40 × 100) cm = 4000 cm.

Convert the data in cm

The length of cloth required to stitch a shirt = 2 m 15 cm

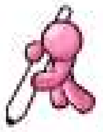
= (2 × 100 + 15) cm = 215 cm.

Dividing 4000 by 215, we get

quotient = 18 and remainder = 130.

Hence, the number of shirt that can be stitched = 18 and the length of the remaining cloth = 130 cm = 1 m 30 cm.

$$\begin{array}{r|l} & 18 \\ 215 & 4000 \\ & -215 \\ \hline & 1850 \\ & -1720 \\ \hline & 130 \end{array}$$



Exercise 1.3

1. In a particular year, a company manufactured 8570435 bicycles and next year it manufactured 8756430 bicycles. In which year more bicycles were manufactured and by how many?
2. What number must be subtracted from 1,02,59,756 to get 77,63,835?
3. The sale receipt of a company during a year was ₹ 30587850. Next year it increased by ₹ 6375490. What was the total sale receipt of the company during these two years?
4. A machine manufactures 23875 screws per day. How many screws did it produce in the year 2012? Assume that the machine worked on all the days of the year. **Hint:** 2012 was a leap year.
5. A merchant had ₹78,592 with him. He placed an order for purchasing 54 bicycles at ₹970 each. How much money will remain with him after the purchase?
6. Amitabh is 1 m 82 cm tall and his wife is 35 cm shorter than him. What is his wife's height?
7. The mass of each gas cylinder is 21 kg 270 g. What is total mass of 28 such cylinders?
8. In order to make a shirt, 2 m 25 cm cloth is needed. What length of cloth is required to make 18 such shirts?
9. The total mass of 12 packets of sweets, each of the same size, is 15 kg 600 g. What is the mass of each such packet?
10. A vessel has 4 litres 500 millilitres of orange juice. In how many glasses, each of 25 mL capacity, can it be filled?
11. To stitch a trouser, 1 m 30 cm cloth is needed. Out of 25 m cloth, how many trousers can be stitched and how much cloth will remain?

ESTIMATION

Estimation is very useful in our daily life. Everyday we hear statements like:

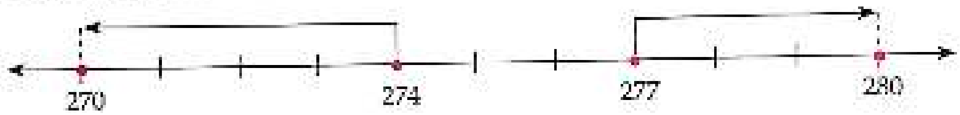
1. Twenty five hundred people enjoyed Salman's movie in a theatre.
2. Sixty seven thousand people watched Sachin Tendulkar's 100th century in the stadium.
3. 13 million passengers cover 63,000 km of railway track everyday.

In fact, no one counted the exact number of people for the particular event. The numbers are estimates giving the idea of the number of people present for the event. The exact counts may be slightly more or less than the estimates. Usually, the numbers are estimated to the nearest tens, hundreds, thousands, and so on.

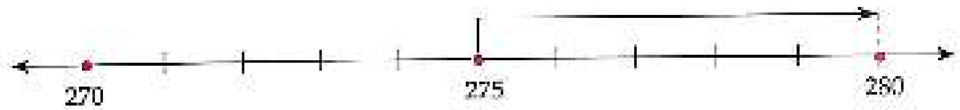
The estimation of numbers is also known as **rounding off numbers.**

Rounding off a number to the nearest tens

Consider the numbers 274 and 277. If we plot these numbers on the number line we find that 274 lies between 270 and 280. Further, we note that 274 is nearer to 270 than to 280. So, we round off 274 as 270, correct to the nearest tens. Also, we find that 277 lies between 270 and 280, and we note that 277 is nearer to 280 than to 270. So, we round off 277 as 280, correct to the nearest tens.



Now consider the number 275. It lies exactly half-way between 270 and 280. By convention we round off 275 as 280.



As the numbers 1, 2, 3 and 4 are nearer to 0 than 10, so we round off 1, 2, 3 and 4 as 0. As the numbers 6, 7, 8 and 9 are nearer to 10 than 0, so we round off 6, 7, 8 and 9 as 10. Further, as the number 5 lies exactly half-way between 0 and 10, by convention, we round off 5 as 10.

Thus, for rounding off a number to the nearest tens, we adopt the following procedure:

Step 1. Look at the digit at ones place of the given number.

Step 2. If the digit at ones place is less than 5, then replace the ones digit by 0 and keep all other digits of the number as they are.

Step 3. If the digit at ones place is 5 or greater than 5, then increase the tens digit by 1 and replace the ones digit by 0.

Example 1. *Round off each of the following numbers to their nearest tens:*

(*i*) 68 (*ii*) 373 (*iii*) 5605 (*iv*) 93695

Solution.

(*i*) The given number is 68.

The digit at ones place is 8, which is greater than 5. So we increase the tens digit by 1 and replace the ones digit by 0.

The rounded off number to the nearest tens = 70.

(*ii*) The given number is 373.

The digit at ones place is 3, which is less than 5. So, we replace the ones digit by 0 and keep all other digits of the number as they are.

The rounded off number to the nearest tens = 370.

(*iii*) The given number is 5605.

The digit at ones place is 5. So, we increase the tens digit by 1 and replace the ones digit by 0.

The rounded off number to the nearest tens = 5610.

(*iv*) The given number is 93695.

The digit at ones place is 5. So, we increase the tens digit by 1 and replace the ones digit by 0.

The rounded off number to the nearest tens = 93700.

Note

Here, the tens digit is 9. So, if we increase this digit by 1 then its effect falls on hundreds digit.

Rounding off a number to the nearest hundreds

Procedure:

Step 1. Look at the digit at tens place in the given number.

Step 2. If the digit at tens place is less than 5, then replace each of the digits at tens place and ones place by 0. Keep all other digits of the number as they are.

Step 3. If the digit at tens place is 5 or greater than 5, then increase the digit at hundreds place by 1 and replace each of the digits at tens place and ones place by 0.

Example 2. *Round off each of the following numbers to their nearest hundreds:*

(*i*) *8532* (*ii*) *23577* (*iii*) *199953*

Solution.

(*i*) The given number is 8532.

The digit at tens place is 3, which is less than 5. So, we replace each of the digits at tens place and ones place by 0. Keep all other digits of the number as they are.

The rounded off number to the nearest hundreds = 8500.

(*ii*) The given number is 23577.

The digit at tens place is 7, which is greater than 5. So, we increase the digit at hundreds place by 1 and replace each of the digits at tens place and ones place by 0.

The rounded off number to the nearest hundreds = 23600.

(*iii*) The given number is 199953.

The digit at tens place is 5. So, we increase the digit at hundreds place by 1 and replace each of the digit at tens place and ones place by 0.

The rounded off number to the nearest hundreds = 200000.

Note

Here, the hundreds digit is 9. So, if we increase this digit by 1 then its effect falls on the digits at higher places.

Rounding off a number to the nearest thousands

Procedure:

Step 1. Look at the digit at hundreds place in the given number.

Step 2. If the digit at hundreds place is less than 5, then replace each of the digits at hundreds place, tens place and ones place by 0. Keep all other digits of the number as they are.

Step 3. If the digit at hundreds place is 5 or greater than 5, then increase the digit at thousands place by 1 and replace each of the digits at hundreds place, tens and ones place by 0.

Example 3. *Estimate each of the following numbers to their nearest thousands:*

(*i*) *5379* (*ii*) *28809* (*iii*) *360516.*

Solution.

(*i*) The given number = 5379.

The digit at hundreds place is 3, which is less than 5.

∴ The estimation of the given number to its nearest thousands = 5000.

(*ii*) The given number is 28809.

The digit at hundreds place is 8, which is greater than 5.

∴ The estimation of the given number to its nearest thousands = 29000.

(*iii*) The given number = 360516

The digit at hundreds place is 5.

∴ The estimation of the given number to its nearest thousands = 361000.

Estimation of sum, difference and product

There are no rigid rules to estimate a number. We can round off a number to any place (tens, hundreds, thousands etc.) depending upon the degree of accuracy required. The most important aspect of the estimation is that the estimated number should make sense *i.e.* the estimation should be reasonable (near to the actual answer).

Example 4. *Estimate the sum 12,904 + 2,888 by estimating the numbers to their nearest:*

(*i*) *tens* (*ii*) *hundreds* (*iii*) *thousands*

Solution. The given numbers are 12,904 and 2,888.

(*i*) Estimating the given numbers to their nearest tens, we get 12,900 and 2,890.

∴ The estimated sum = 15,790.

$$\begin{array}{r} 12{,}900 \\ +\ 2{,}890 \\ \hline 15{,}790 \end{array}$$

(*ii*) Estimating the given numbers to their nearest hundreds, we get 12,900 and 2,900.

∴ The estimated sum = 15,800.

$$\begin{array}{r} 12{,}900 \\ +\ 2{,}900 \\ \hline 15{,}800 \end{array}$$

(*iii*) Estimating the given numbers to their nearest thousands, we get 13,000 and 3,000.

∴ The estimated sum = 16,000.

$$\begin{array}{r} 13{,}000 \\ +\ 3{,}000 \\ \hline 16{,}000 \end{array}$$

Example 5. *Estimate: 5673 – 436 by estimating the numbers to their nearest*

(*i*) *thousands* (*ii*) *hundreds* (*iii*) *greatest places.*

Also, point out the most reasonable estimate.

Solution. The given numbers are 5673 and 436.

(*i*) Estimating the given numbers to their nearest thousands, we get, 6000 and 0.

Estimated difference = 6000 – 0 = 6000.

(*ii*) Estimating the given numbers to their nearest hundreds, we get, 5700 and 400.

Estimated difference = 5700 – 400 = 5300.

(*iii*) The estimation of the number 5673 nearest to its greatest place *i.e.* the thousands place = 6000.

The estimation of the number 436 nearest to its greatest place *i.e.* the hundreds place = 400.

Estimated difference = 6000 – 400 = 5600.

Actual difference = 5673 – 436 = 5237.

Thus, we find that the estimation of the given numbers to their nearest hundreds is the most reasonable estimate.

Example 6. *Estimate the product: 81 × 479 by rounding off each factors nearest to its greatest place.*

Solution. Rounding off 81 to its greatest place *i.e.* tens place, estimated number = 80.

Rounding off 479 to its greatest place *i.e.* hundreds place, estimated number = 500.

∴ Estimated product = 80 × 500 = 40000.

Example 7. *Estimate the product: 1291 × 592 by rounding off each factor to its*

(*i*) *greatest place* (*ii*) *nearest hundreds.*

Solution. The given numbers are 1291 and 592.

(*i*) Rounding off 1291 to its greatest place *i.e.* thousands place,

estimated number = 1000.

Rounding off 592 to its greatest place *i.e.* hundreds place,

estimated number = 600.

Estimated product = 1000 × 600 = 6,00,000.

(*ii*) Rounding off 1291 to its nearest hundreds, estimated number = 1300.

Rounding off 592 to its nearest hundreds, estimated number = 600.

Estimated product = 1300 × 600 = 7,80,000.

Note that 1291 × 592 = 7,64,272 (obtain it)

Thus, we note that the estimation according to rounding off to nearest hundreds is more reasonable.

Exercise 1.4

1. Round off each of the following numbers to their nearest tens:
   (*i*) 77 (*ii*) 903 (*iii*) 1205 (*iv*) 999
2. Estimate each of the following numbers to their nearest hundreds:
   (*i*) 1246 (*ii*) 32057 (*iii*) 53961 (*iv*) 555555
3. Estimate each of the following numbers to their nearest thousands:
   (*i*) 5706 (*ii*) 378 (*iii*) 47,599 (*iv*) 1,09,736
4. Give a rough estimate (by rounding off to nearest hundreds) and also a closer estimate (by rounding off to nearest tens):
   (*i*) 439 + 334 + 4317 (*ii*) 8325 – 491
   (*iii*) 1,08,734 – 47,599 (*iv*) 4,89,348 – 48,365
5. Estimate each of the following by rounding off each number nearest to its greatest place:
   (*i*) 730 + 998 (*ii*) 5,290 + 17,986
   (*iii*) 796 – 314 (*iv*) 28,292 – 21,496
6. Estimate the following products by rounding off each of its factors nearest to its greatest place:
   (*i*) 578 × 161 (*ii*) 9650 × 27
7. Estimate the following products by rounding off each of its factors nearest to its hundreds place:
   (*i*) 5281 × 3491 (*ii*) 1387 × 888

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (*i*) The digit ... has the highest place value in the number 2309.
   (*ii*) The digit ... has the highest face value in the number 2039.
   (*iii*) The digit ... has the lowest place value in the number 2039.
   (*iv*) Both Indian and International systems of numeration have ... period in common.
   (*v*) In the International system of numeration, commas are placed from ... after every ... digits.
   (*vi*) The bigger number from the numbers 57,631 and 57,361 is ....
   (*vii*) 1 crore = ... million
   (*viii*) The smallest 4-digit number with 3 different digits is ...
   (*ix*) The greatest 4-digit number with 3 different digits is ...
   (*x*) 15 km 300 m = ... m
   (*xi*) 7850 cm = ... m ... cm
   (*xii*) The number 5079 when estimated to the nearest hundreds is ...

2. State whether the following statements are true (T) or false (F):
   (i) The difference between the place value and the face of the digit 7 in the number 2701 is 693.
   (ii) The smallest 4-digit number – 1 = the greatest 3-digit number.
   (iii) The place of a digit is independent of whether the number is written in the Indian system or International system of numeration.
   (iv) In the International system, a number having less number of digits is always smaller than the number having more number of digits.
   (v) The estimated value of 9999 to the nearest tens is 10000.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 17):

3. The face value of the digit 5 in the number 36,503 is
   (*a*) 5 (*b*) 503 (*c*) 500 (*d*) none of these
4. The difference between the place values of 6 and 3 in 76834 is
   (*a*) 3 (*b*) 5700 (*c*) 5930 (*d*) 5970
5. The sum of the place values of all the digits in 5003 is
   (*a*) 8 (*b*) 53 (*c*) 5003 (*d*) 8000
6. The total number of 4-digit numbers is
   (*a*) 9000 (*b*) 9999 (*c*) 10000 (*d*) none of these
7. The product of the place values of two-threes in 73532 is
   (*a*) 9000 (*b*) 90000 (*c*) 99000 (*d*) 1000
8. The smallest 4-digit number having distinct digits is
   (*a*) 1234 (*b*) 1023 (*c*) 1002 (*d*) 3210
9. The largest 4-digit number having distinct digits is
   (*a*) 9999 (*b*) 9867 (*c*) 9786 (*d*) 9876
10. The largest 4-digit number is
    (*a*) 9999 (*b*) 9876 (*c*) 9990 (*d*) none of these
11. The difference between the largest number of 3-digit and the largest number of 3-digit with distinct digits is
    (*a*) 0 (*b*) 10 (*c*) 12 (*d*) 14
12. If we write numbers from 1 to 100, the number of times the digit 5 has been written is
    (*a*) 11 (*b*) 15 (*c*) 19 (*d*) 20
13. The number 28,549 when rounded off to the nearest hundreds is
    (*a*) 28,000 (*b*) 28,500 (*c*) 28,600 (*d*) 29,000
14. The smallest natural number which when rounded off to the nearest hundreds as 500 is
    (*a*) 499 (*b*) 501 (*c*) 450 (*d*) 549
15. The greatest natural number which when rounded off to the nearest hundreds as 500 is
    (*a*) 549 (*b*) 599 (*c*) 450 (*d*) none of these
16. The greatest 5-digit number formed by the digits 3, 0, 7 is
    (*a*) 33077 (*b*) 77730 (*c*) 77330 (*d*) none of these

17. In the International place value system, we write 1 billion for
(*a*) 10 lakh (*b*) 1 crore (*c*) 10 crore (*d*) 100 crore

Value Based Question

The distance between Anu's home and her school is 4 km 85 m. Everyday she cycles both ways. Find the distance covered by her in a week. (Sunday being a holiday).
What are the advantages of cycling?

Higher Order Thinking Skills (HOTS)

1. Is there any digit whose place value is always equal to its face value irrespective of its position in any number?
2. Write all 4-digit numbers that can be formed with the digits 2 and 5, using both digits equal number of time. Also find their sum.
3. What is the difference between the smallest 6-digit number with five different digits and the greatest 5-digit number with four different digits?
4. How many times does the digit 3 occur at ten's place in natural numbers from 100 to 1000?

Summary

★ Counting numbers 1, 2, 3,... are called **natural numbers**.
★ 1 is the smallest natural number.
★ There is no largest natural number.
★ Any number (however large) can be written by using ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called **digits or figures**.
★ The place **value** of a (non-zero) digit depends upon the place it occupies in the number; the place value of the digit 0 is always 0 regardless of the place it occupies in the number.
★ The **face value** of a digit in a number is the digit itself, regardless of the place it occupies in the number.
★ If a place is vacant in a number, put 0 at that place.
★ Never put 0 as the extreme left digit of a number.
★ A single digit or a group of digits representing a number is called **numeral.**
★ Writing a number in words is called **numeration.**
★ To read and write numbers, the two system of numeration in common use are:
(*i*) Indian system (*ii*) International system.
★ In Indian system, the various periods from the right are:
ones, thousands, lakhs, crores and so on.
While in International system, periods from the right are:
ones, thousands, millions, billions and so on.

★ In Indian system, the first comma comes after 3-digits, from the right and next comma comes after every 2-digits; while in International system, the commas come after every 3-digits from the right.

★ 1 million = 10 lakh, 10 million = 1 crore, 100 million = 10 crore, 1 billion = 100 crore.

★ **Comparison of numbers.** Given two numbers, the number having more digits is greater. If the number of digits is equal, then start comparing digits from the extreme left *i.e.* the highest place till we get a pair of unequal digits, the number having greater digit is greater.

★ 1 km = 1000 m, 1 m = 100 cm, 1 cm = 10 mm
1 kg = 1000 g, 1 g = 1000 mg
1 kL = 1000 L, 1 L = 1000 mL

★ **Estimation.** To estimate (or round off) a number to the nearest

★ **Tens:**
(*i*) If the digit at ones place is less than 5, then replace ones digit by 0 and keep all other digits as they are.
(*ii*) If the digit at ones place is 5 or greater than 5, then increase the tens digits by 1 and replace the ones digits by 0.

★ **Hundreds:**
(*i*) If the digit at tens place is less than 5, then replace each of the digits at tens place and ones place by 0. Keep all other digits as they are.
(*ii*) If the digit at tens place is 5 or greater than 5, then increase the digit at hundreds place by 1 and replace each of the digits at tens place and ones place by 0.

★ **Thousands:**
(*i*) If the digit at hundreds place is less than 5, then replace each of the digits at hundreds place, tens place and ones place by 0. Keep all other digits as they are.
(*ii*) If the digit at hundreds place is 5 or greater than 5, then increase the digit at thousands place by 1 and replace each of the digits at hundreds place, tens place and ones place by 0.

Check Your Progress

1. Write the numeral for each of the following numbers and insert commas correctly:
   (*i*) Six crore nine lakh forty seven.
   (*ii*) One hundred four million seven hundred twenty two thousand three hundred ninety four.
2. Insert commas suitably and write the number 30189301 in words in Indian and International system of numeration.
3. Find the difference between the place value and the face value of the digit 6 in the number 72601.
4. Write all possible two-digit numbers using the digits 4 and 0. Repetition of digits is allowed.
5. Write all possible natural numbers using the digits 7, 0, 6. Repetition of digits is not allowed.

[**Hint.** Here, the numbers can be of one, two or three digits.]

6. Arrange the following numbers in ascending order:
3706, 58019, 3760, 59801, 560023

7. Write the greatest six-digit number using four different digits.

8. Write the smallest eight-digit number using four different digits.

9. Find the difference between the greatest and the smallest 4-digit numbers formed by the digits 0, 3, 6, 9.

10. Find the sum of the four-digit greatest number and the five-digit smallest number, each number having three different digits.

11. Write the greatest and the smallest four-digit numbers using four different digits with the conditions as given:

(*i*) Digit 3 always at hundred's place.

(*ii*) Digit 0 always at ten's place.

12. A mobile number consists of ten digits. First four digits are 9, 9, 7 and 9. Make the smallest mobile number by using only one digit twice from the digits 8, 3, 5, 0, 6.

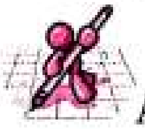
13. To stitch a uniform, 1 m 75 cm cloth is needed. Out of 153 m cloth, how many uniforms can be stitched and how much cloth will remain?

14. Medicine is packed in boxes, each weighing 4 kg 500 g. How many such boxes can be loaded in a van which cannot carry beyond 800 kg?

15. Estimate: 6554 – 677 by estimating the numbers to their nearest

(*i*) thousands (*ii*) hundreds (*iii*) greatest places

Also point out the most reasonable estimate.



Activity 1

Objective

To round off the numbers which represent population and area of different states according to the latest census (2011).

Pre-required knowledge

Concept of rounding off the numbers.

Materials required

White sheets of paper
Coloured pens

To perform activity

A. Rounding off number to the nearest hundreds of given numbers.

Procedure

1. Look at the digit at tens place in the given number.
2. If the digit at tens place is less than 5, then replace each of the digits at tens place and once place by 0. Keep all other digits of the numbers as they are.
3. If the digit at tens place is 5 or greater than 5, then increase the digit at hundreds place by 1 and replace each of the digits at tens place and ones place by 0.

2 WHOLE NUMBERS

INTRODUCTION

The concept of the number 'zero' came much later than that of the natural numbers.

Suppose, Ankita has five toffees and she has eaten up all the five, so she is left with no toffees. Another way of saying this is that Ankita is left with zero toffees. The number zero (which Indians call 'Shunya' and Arabs called '**Cipher**') is denoted by the symbol '0'. It is believed that the number zero was first introduced by Aryabhata.

In this chapter, you will learn :

- Whole numbers
- Representation of whole numbers on number line
- Successor and predecessor of a whole number
- Properties of addition
- Properties of multiplication
- Operation of division
- Division algorithm
- Properties of division
- Patterns in whole numbers.

ARYABHATA

Aryabhata (Sanskrit-आर्यभट) was the first in the line of great mathematician-astronomers from the classical age of Indian mathematics and Indian astronomy. His most famous works are the ***Aryabhatiya*** (499 A.D., when he was 23 years old) and the ***Arya-sidhanta***. Aryabhata provides no information about his place of birth. It is believed that he went to Kusumapura (Patliputra, modern Patna) for advanced studies at the University of Nalanda (Bihar), which was then a great place of learning.

ARYABHATA
(476 – 520)

Works

Aryabhata was the author of several treatises on mathematics and astronomy, some of which are lost. However, his major work 'Aryabhatiya' has survived to modern times. The mathematical part of *Aryabhatiya* covers arithmetic, algebra, plane trigonometry and spherical trigonometry. It also contains continued fractions, quadratic equations, sums of power series and a table of sines.

Place value system and zero

The place value system is clearly in place in his work. The knowledge of zero was implicit in Aryabhata's place value system as a place holder for the powers of ten with null coefficients.

India's first satellite Aryabhata and the lunar crater Aryabhata are named in his honour. The inter-school Math Competition is also named after him.

WHOLE NUMBERS

In the previous chapter, we have learnt about counting numbers 1, 2, 3, 4, ... These numbers come naturally when we begin counting. Hence, we call the counting numbers as **natural numbers.** The ***smallest*** natural number is 1 and there is no ***largest*** natural number.

Representation of whole numbers on number line

To represent whole numbers on a line, we proceed as under:

(1) Draw any straight line, mark point O on it and label it as 0 (zero).

(2) Mark another point A to the right of O on the line and label it as 1 (one), then the length of the segment OA = 1 unit.

(3) Mark points B, C, D, E, ... to the right of A at unit length intervals on the line and label these points as 2, 3, 4, 5, ... respectively.

Thus, the points O, A, B, C, D, E, ... on the line represent the whole numbers 0, 1, 2, 3, 4, 5, ...

Continuing the above process, we can represent every whole number by some point on the line.

The line drawn above is called the **number line.**

From the number line, we observe that:

1. *There is no whole number on the left of '0', and every whole number on its right is greater than 0.*

   Thus, 0 is the smallest whole number.

   As the above process of marking points to the right of 0 does not stop anywhere, there is no largest whole number.

2. *A whole number is greater than all those whole numbers that lie to its left on the number line.*

   In other words, a whole number is greater than the other whole number if the first number lies on the right of the second number on the number line.

   For example:

   As 5 lies on the right of 2 on the number line, so $5 > 2$.

   Similarly, $7 > 3$ and $11 > 10$.

3. *A whole number is less than all those whole numbers that lie to its right on the number line.*

   In other words, a whole number is less than the other whole number if the first number lies on the left of the second number on the number line.

   For example:

   As 2 lies on the left of 5 on the number line, so $2 < 5$.

   Similarly, $3 < 7$ and $10 < 11$.

4. *There is no whole number between any two consecutive whole numbers; and there is atleast one whole number between two non-consecutive whole numbers.*

Consecutive means that the numbers are next to one-another.

For example:

(*i*) There is no whole number between two consecutive whole numbers 5 and 6.

(*ii*) The whole number 5 lies between two non-consecutive whole numbers 4 and 6.

Obviously $4 < 5$ and $5 < 6$; we may write $4 < 5 < 6$.

(*iii*) The whole numbers 5 and 7 lie between two non-consecutive whole numbers 4 and 10.

Obviously $4 < 5 < 10$ and $4 < 7 < 10$.

Successor and predecessor

One more than a given whole number is called its ***successor.***

Thus, 1 is the successor of 0, 2 is the successor of 1 and so on.

Note that each whole number has one and only one successor; and it is the successor of the whole number just on its left on the number line. Note that 0 is not the successor of any whole number.

One less than a given whole number (other than zero) is called its ***predecessor.***

Thus, 0 is the predecessor of 1, 1 is the predecessor of 2 and so on.

Note that each whole number (except 0) has one and only one predecessor; and it is the predecessor of the whole number just on its right on the number line.

Note that 0 has no predecessor in the whole number system.

Remarks

* Successor of a whole number = (given whole number) + 1.
* Predecessor of a whole number = (given whole number) – 1.
* 0 is not the successor of any whole number.
* 0 has no predecessor in whole number system.
* 1 has no predecessor in natural number system.

Example 1. *Write the successor of each of the following numbers:*

(*i*) *3799* (*ii*) *531010*

Solution. Successor is 1 more than the given number.

(*i*) The successor of 3799

$= 3799 + 1 = 3800.$

(*ii*) The successor of 531010

$= 531010 + 1 = 531011.$

Successor is 1 more than the given number

Example 2. *Write the predecessor of each of the following numbers:*

(*i*) *3799* (*ii*) *531010*

Solution. Predecessor is 1 less than the given number.

(*i*) The predecessor of 3799
$= 3799 - 1 = 3798$.

(*ii*) The predecessor of 531010
$= 531010 - 1 = 531009$.

> Predecessor is 1 less than the given number

Example 3. *Write the whole number whose successor is 379600.*

Solution. The required whole number = predecessor of 379600
$= 379600 - 1 = 379599$.

Example 4. *Write the whole number whose predecessor is 74999.*

Solution. The required whole number = successor of 74999
$= 74999 + 1 = 75000$.

Example 5. *Write two whole numbers occurring just before 10001.*

Solution. Two whole numbers occurring just before 10001 are
$10001 - 1$ and $10001 - 2$ *i.e.* 10000 and 9999.

Example 6. *Write the next three consecutive whole numbers of the following numbers:*

(*i*) *59* (*ii*) *37898*

Solution.

(*i*) The next three consecutive whole numbers of 59 are:
60, 61, 62.

(*ii*) The next three consecutive whole numbers of 37898 are:
37899, 37900, 37901.

Example 7. *How many whole numbers are there between 81 and 101?*

Solution. The whole numbers between 81 and 101 are:
82, 83, 84, ..., 100.

Number of these numbers $= 100 - 81 = 19$.

Note that in the above list, 100 is included and 81 is not included.

Example 8. *How many 3-digit numbers are there between 94 and 607?*

Solution. 3-digit numbers between 94 and 607 are:
100, 101, 102, ... 606.

Number of these numbers $= 606 - 99 = 507$.

Exercise 2.1

1. Write the smallest whole number. Can you write the largest whole number?
2. Write the successor of each of the following numbers:
   (*i*) 3999 (*ii*) 378915 (*iii*) 5001299.

3. Write the predecessor of each of the following numbers:
   (*i*) 500 (*ii*) 38794 (*iii*) 54789011.
4. Write the whole number (in each of the following) whose successor is:
   (*i*) 50795 (*ii*) 720300 (*iii*) 8300000.
5. Write the whole number (in each of the following) whose predecessor is:
   (*i*) 5347 (*ii*) 72399 (*iii*) 3012999.
6. Write next three consecutive whole numbers of the following numbers:
   (*i*) 79 (*ii*) 598 (*iii*) 35669
7. Write three consecutive whole numbers occurring just before 320001.
8. (*i*) How many whole numbers are there between 38 and 68?
   (*ii*) How many whole numbers are there between 99 and 300?
9. Write all whole numbers between 100 and 200 which do not change if the digits are written in reverse order.
10. How many 2-digit whole numbers are there between 5 and 92?
11. How many 3-digit whole numbers are there between 72 and 407?

FUNDAMENTAL OPERATIONS ON WHOLE NUMBERS

We are already familiar with the four fundamental operations of ***addition, subtraction, multiplication*** and ***division*** on the whole numbers. In this section, we shall learn some basic properties of the operations of addition and multiplication on the whole numbers. These properties help us to understand the numbers better and sometimes make calculations very simple. We shall also review the operations of subtraction and division on whole numbers.

Properties of addition

- **Closure property of addition**

Let us add any two whole numbers and check whether the sum is a whole number.

Whole number	*Whole number*	*Sum*	*Is the sum a whole number?*
18	23	18 + 23 = 41	Yes
127	308	127 + 308 = 435	Yes
239	239	239 + 239 = 478	Yes

Thus, we find that the sum of any two whole numbers is a whole number. In other words:

If a and b are any two whole numbers, then a + b is also a whole number. This is called *closure property of addition*.

- **Commutative property of addition**

Let us add any two whole numbers in two different orders and check whether the sum is same.

For example:

(*i*) 39 + 28 = 67, 28 + 39 = 67.
Is 39 + 28 = 28 + 39? Yes.

(ii) $231 + 78 = 309$, $78 + 231 = 309$.

Is $231 + 78 = 78 + 231$? Yes.

From the above examples, we find that in whatever order we add any two whole numbers, their sum remains the same.

In other words:

If a and b are any two whole numbers, then $a + b = b + a$. This is called *commutative property of addition.*

- **Associative law of addition**

Let us take any three whole numbers and find the sum of these numbers. We find the sum of any two of them and add their sum to the third number in two different ways of associating them and check whether the result is same.

For example:

(i) $(8 + 13) + 6 = 21 + 6 = 27$ and $8 + (13 + 6) = 8 + 19 = 27$.

Is $(8 + 13) + 6 = 8 + (13 + 6)$? Yes.

(ii) $(52 + 17) + 23 = 69 + 23 = 92$ and $52 + (17 + 23) = 52 + 40 = 92$.

Is $(52 + 17) + 23 = 52 + (17 + 23)$? Yes.

From the above examples, we find that the sum of any three whole numbers in whatever way we associate them, remains the same.

In other words:

If a, b and c are any three whole numbers, then $(a + b) + c = a + (b + c)$. This is called *associative law of addition.*

- **Existence of additive identity**

The number '0' has a special role in addition.

6	+	0	=	6
0	+	6	=	6
23	+	0	=	23
0	+	23	=	23

Look at the adjoining table, we note that: If a is any whole number, then

$$a + 0 = a = 0 + a.$$

The number '0' is called the **additive identity.**

- **Cancellation law of addition**

If a, b and c are any whole numbers, then $a + c = b + c \Rightarrow a = b$.

For example:

If x is a whole number, then $x + 5 = 3 + 5 \Rightarrow x = 3$.

Remarks

- In view of the associative law of addition, to add any three whole numbers we can add any two numbers and then add the sum to the third number. Hence, we can drop parenthesis ( ) and write the sum of three numbers a, b and c as $a + b + c$.
  Thus, $(a + b) + c = a + (b + c) = a + b + c$.
- In view of the commutative property and associative law of addition, we note that while adding any three or more whole numbers we can group them or change their order in such a way that the calculations become easier.

Example 1. *Find the following sum by suitable rearrangement:*

$837 + 509 + 363.$

Solution. $837 + 509 + 363 = (837 + 363) + 509$

$= 1200 + 509 = 1709.$

Example 2. *Find the sum of 123, 254, 37, 105 and 5046.*

Solution. $123 + 254 + 37 + 105 + 5046 = (123 + 37) + (254 + 5046) + 105$

$= 160 + 5300 + 105 = (160 + 5300) + 105$

$= 5460 + 105 = 5565.$

Subtraction in whole numbers

Look at the adjoining table:

8	–3	=	5, a whole number
25	–9	=	16, a whole number
5	–7	=	?, not a whole number

Thus, if a and b are whole numbers such that $a \geq b$, then $a - b$ is a whole number but if $a < b$ then $a - b$ is not a whole number.

Hence, the whole numbers are not closed under subtraction.

In general, if a, b and c are any whole numbers, then

(*i*) $a - b \neq b - a$

(*ii*) $(a - b) - c \neq a - (b - c)$

(*iii*) $a - 0 = a$ but $0 - a$ is not a whole number *i.e.* 0 is not identity under subtraction.

Properties of multiplication

- **Closure property of multiplication**

Let us multiply any two whole numbers and check whether the product is a whole number.

Whole number	*Whole number*	*Product*	*Is the product a whole number?*
12	8	$12 \times 8 = 96$	Yes
23	11	$23 \times 11 = 253$	Yes
57	15	$57 \times 15 = 855$	Yes

Thus, we find that the product of any two whole numbers is a whole number. In other words:

If a and b are any two whole numbers, then $a \times b$ is also a whole number. This is called *closure property of multiplication.*

- **Commutative property of multiplication**

Let us multiply any two whole numbers in two different orders and check whether the product is same.

For example:

(*i*) $7 \times 9 = 63$, $9 \times 7 = 63$.

Is $7 \times 9 = 9 \times 7$? Yes.

(*ii*) $25 \times 16 = 400$, $16 \times 25 = 400$.

Is $25 \times 16 = 16 \times 25$? Yes.

From the above examples, we find that in whatever order we multiply any two whole numbers, their product remains the same.

In other words:

If a and b are any two whole numbers, then $a \times b = b \times a$. This is called commutative property of multiplication.

- **Associative law of multiplication**

  Let us take any three whole numbers and find the product of these numbers. We find the product of any two of them and multiply their product with the third number in two different ways of associating them and check whether the result is same.

  For example:

  (i) $(7 \times 9) \times 5 = 63 \times 5 = 315$ and $7 \times (9 \times 5) = 7 \times 45 = 315$.

  Is $(7 \times 9) \times 5 = 7 \times (9 \times 5)$? Yes.

  (ii) $(12 \times 8) \times 11 = 96 \times 11 = 1056$ and $12 \times (8 \times 11) = 12 \times 88 = 1056$.

  Is $(12 \times 8) \times 11 = 12 \times (8 \times 11)$? Yes.

  From the above examples, we find that the product of any three whole numbers in whatever way we associate them remains the same.

  In other words:

  If a, b and c are any three whole numbers, then $(a \times b) \times c = a \times (b \times c)$. This is called associative law of multiplication.

- **Multiplication is distributive over addition**

  Let us find $7 \times (5 + 11)$ in two different ways.

  $7 \times (5 + 11) = 7 \times 16 = 112$.

  Also $7 \times 5 + 7 \times 11 = 35 + 77 = 112$.

  Thus $7 \times (5 + 11) = 7 \times 5 + 7 \times 11$.

  Similarly, we can verify that

  $12 \times (15 + 18) = 12 \times 15 + 12 \times 18$.

  From the above examples, it follows that:

  If a, b and c are any three whole numbers then $a \times (b + c) = a \times b + a \times c$. This is known as distributive law of multiplication over addition.

  Think and write

  Observe the following multiplication and think whether we have used the idea of distributivity of multiplication over addition:

$$
\begin{array}{r l}
437 & \\
\times\ 253 & \\
\hline
1311 & \leftarrow 437 \times 3 \\
21850 & \leftarrow 437 \times 50 \\
87400 & \leftarrow 437 \times 200 \\
\hline
110561 & \leftarrow 437 \times (3 + 50 + 200)
\end{array}
$$

In fact, the process of multiplication (shown above) which we were using in our earlier classes is based on the property of distribution of multiplication over addition.

- **Existence of multiplicative identity**

The number '1' has a special role in multiplication.

Look at the adjoining table. We note that:

7	×	1	=	7
1	×	7	=	7
38	×	1	=	38
1	×	38	=	38

If a is any whole number, then

$a \times 1 = a = 1 \times a.$

The number '1' is called the multiplicative identity.

- **Multiplication by zero**

The number zero has a special role in multiplication.

Look at the adjoining pattern:

5	×	4	=	20
5	×	3	=	15
5	×	2	=	10
5	×	1	=	5
5	×	0	=	?

We note that at each step the product decreases by 5. So, the last product should also decrease by 5 *i.e.* $5 \times 0 = 5 - 5 = 0$.

In fact, this is true for all other whole numbers also.

Thus, if a is any whole number, then $a \times 0 = 0 = 0 \times a$.

- **Cancellation law of multiplication**

If a and b are any whole numbers and c is a non-zero whole number, then

$a \times c = b \times c \Rightarrow a = b.$

For example:

If x is a whole number, then

$5 \times x = 15 \Rightarrow 5 \times x = 5 \times 3 \Rightarrow x = 3.$

Remarks

✱ In view of the associative law of multiplication, to multiply any three whole numbers we can multiply any two numbers and then multiply this product with the third number. Hence, we can drop parenthesis ( ) and write the product of three numbers a, b and c as $a \times b \times c$. Thus,

$(a \times b) \times c = a \times (b \times c) = a \times b \times c.$

✱ In view of the commutative property and associative law of multiplication, we note that while multiplying any three or more whole numbers we can group them or change their order in such a way that the calculations become easier.

- **Multiplication is distributive over subtraction**

Let us find $6 \times (13 - 5)$ in two different ways.

$6 \times (13 - 5) = 6 \times 8 = 48.$

Also $6 \times 13 - 6 \times 5 = 78 - 30 = 48.$

From the above example, it follows that:

If a, b, c are whole numbers and $b \geq c$, *then*

$a \times (b - c) = a \times b - c \times c.$

This is known as *distributive law of multiplication over subtraction.*

Example 3. *Find the following products by suitable rearrangements:*

(i) $8 \times 367 \times 25$ (ii) $17 \times 125 \times 14 \times 8$.

Solution. (*i*) $8 \times 367 \times 25 = 367 \times (8 \times 25)$
$= 367 \times 200 = 73400.$

(*ii*) $17 \times 125 \times 14 \times 8 = (17 \times 14) \times (125 \times 8)$
$= 238 \times 1000 = 238000.$

Example 4. *Find the value of the following:*

(*i*) $658 \times 42 + 658 \times 158$ (*ii*) $81265 \times 187 - 81265 \times 87.$

Solution. (*i*) $658 \times 42 + 658 \times 158 = 658 \times (42 + 158)$
$= 658 \times 200 = 131600.$

(*ii*) $81265 \times 187 - 81265 \times 87 = 81265 \times (187 - 87)$
$= 81265 \times 100 = 8126500.$

Example 5. *Using suitable properties, find the following products:*

(*i*) 674×110 (*ii*) 1396×99 (*iii*) $1006 \times 178.$

Solution. (*i*) $674 \times 110 = 674 \times (100 + 10) = 674 \times 100 + 674 \times 10$
$= 67400 + 6740 = 74140.$

(*ii*) $1396 \times 99 = 1396 \times (100 - 1) = 1396 \times 100 - 1396 \times 1$
$= 139600 - 1396 = 138204.$

(*iii*) $1006 \times 178 = (1000 + 6) \times (100 + 78)$
$= 1000 \times 100 + 1000 \times 78 + 6 \times 100 + 6 \times 78$
$= 100000 + 78000 + 600 + 468 = 179068.$

OPERATION OF DIVISION

Let us divide 30 by 5.

Dividing 30 by 5 is same as finding a whole number which when multiplied by 5 gives 30. Certainly, it is 6 because $5 \times 6 = 30$. Therefore, the statement $30 \div 5 = 6$ is another way of writing the statement $5 \times 6 = 30$.

Let a and b ($\neq 0$) be two whole numbers, then dividing a by b is same as finding a whole number c which when multiplied by b gives a *i.e.* $b \times c = a$.

Thus, $a \div b = c$ is same as $b \times c = a$.

Hence, operation of division is an inverse operation of multiplication.

Division by zero

We know that multiplication by a (natural) number means adding that number repeatedly. As division is an inverse operation of multiplication, so division by a number means subtracting that number repeatedly.

Let us find $12 \div 4$

```
 12
− 4  ←—— move 1
 ──
  8
− 4  ←—— move 2
 ──
  4
− 4  ←—— move 3
 ──
  0
```

Subtract 4 from 12 again and again. Note that after 3 moves, we reach 0.

So, we write $12 \div 4 = 3$.

Similarly, $20 \div 5 = 4$, $42 \div 7 = 6$ etc.

Now, let us try $3 \div 0$

$$\begin{array}{rl} 3 & \\ -0 & \leftarrow \text{move 1} \\ \hline 3 & \\ -0 & \leftarrow \text{move 2} \\ \hline 3 & \\ -0 & \leftarrow \text{move 3} \\ \hline 3 & \end{array}$$

Note that after every move we get 3 again. So, this process will never stop *i.e.* we never reach 0.
We say that $3 \div 0$ is not defined.
Thus, division of a whole number by 0 is not defined.

Alternatively

Dividing 3 by 0 is the same as finding a whole number, say b, which when multiplied by 0 gives 3. But $0 \times b = 0$ *i.e.* there does not exist any whole number b such that $0 \times b = 3$. We say that $3 \div 0$ is not defined. Thus, if a is any (non-zero) whole number then $a \div 0$ is not defined.

Hence, the division of a whole number by 0 is not defined.

Division algorithm

We already know how to divide a whole number by another (non-zero) smaller whole number. Let us see what happens on dividing 31 by 7. We have:

$$\begin{array}{r} 7\,)\,31\,(\,4 \\ -28 \\ \hline 3 \end{array}$$

Note that $31 = 7 \times 4 + 3$.

The number (here 31) which is to be divided is called ***dividend***. The number (here 7) by which the dividend is divided is called ***divisor***. The number of times (here 4) the divisor is contained in the dividend is called ***quotient***. The number (here 3) which is left over after division is called ***remainder***. In the above example, we see that

dividend = divisor × quotient + remainder

This result is true and can be verified by means of examples. Thus, we have:

If a is any whole number and b is another smaller non-zero whole number then there exist unique whole numbers q and r such that

$a = b \times q + r$ *where* $0 \le r < b$.

This is called ***division algorithm*** or ***division rule***.

Properties of division

Look at the adjoining table:

24	÷	6	=	4, a whole number
35	÷	7	=	5, a whole number
21	÷	5	=	?, not a whole number

Thus, if a and b ($\neq 0$) are whole numbers such that $a \ge b$, then $a \div b$ may not be a whole number.

Hence, the whole numbers are not closed under division.

- ***If a is any whole number, then $a \div 1 = a$.***

  Justification: As $1 \times a = a$, so $a \div 1 = a$.

- *If a is any (non-zero) whole number, then a ÷ a = 1*

  Justification: As $a \times 1 = a$, so $a \div a = 1$.
- *If a is any (non-zero) whole number, then 0 ÷ a = 0.*

  Justification: As $a \times 0 = 0$, so $0 \div a = 0$.

Example 6. *Divide 279 by 13 and check the result by division algorithm.*

Solution. By actual division, we have

Dividend = 279, divisor = 13, quotient = 21

and remainder = 6.

Check: divisor × quotient + remainder

= 13 × 21 + 6 = 273 + 6 = 279

= dividend

Hence, the result is correct.

```
13 ) 279 ( 21
    -26
     19
    -13
      6
```

Example 7. *Find the greatest 4-digit number which is exactly divisible by 135.*

Solution. The greatest 4-digit number = 9999.

We divide 9999 by 135 and find the remainder.

∴ The smallest number which should be subtracted from 9999 so that the remaining number is exactly divisible by 135 is 9.

∴ The required number = 9999 – 9 = 9990.

```
135 ) 9999 ( 74
     -945
      549
     -540
        9
```

Example 8. *Find the least number which should be added to 10000 so that the sum is exactly divisible by 237.*

Solution. We divide 10000 by 237 and find the remainder.

∴ The least number which should be added to 10000 so that the sum is exactly divisible by 237 is 237 – 46 *i.e.* 191

Note that 10000 + 191 *i.e.* 10191 is the smallest 5-digit number which is exactly divisible by 237.

```
237 ) 10000 ( 42
      -948
       520
      -474
        46
```

Exercise 2.2

1. Fill in the blanks to make each of the following a true statement:
   (*i*) 378 + 1024 = 1024 + ...
   (*ii*) 337 + (528 + 1164) = (337 + ...) + 1164
   (*iii*) (21 + 18) + ... = (21 + 13) + 18
   (*iv*) 3056 + 0 = ... = 0 + 3056
2. Add the following numbers and check by reversing the order of addends:
   (*i*) 3189 + 53885 (*ii*) 33789 + 50311.
3. By suitable arrangements, find the sum of:
   (*i*) 311, 528, 289 (*ii*) 723, 834, 66, 277 (*iii*) 78, 203, 435, 7197, 422.
4. Fill in the blanks to make each of the following a true statement:
   (*i*) 375 × 57 = 57 × .... (*ii*) (33 × 16) × 25 = 33 × (.... × 25)

(iii) $37 \times 24 = 37 \times 18 + 37 \times$ .... (iv) $7205 \times 1 = ... = 1 \times 7205$

(v) $366 \times 0 =$ .... (vi) ... $\times 579 = 0$

(vii) $473 \times 108 = 473 \times 100 + 473 \times$ ... (viii) $684 \times 97 = 684 \times 100 -$ ... $\times 3$

(ix) $0 \div 5 =$ ... (x) $(14 - 14) \div 7 =$ ....

5. Determine the following products by suitable arrangement:
   (i) $4 \times 528 \times 25$ (ii) $625 \times 239 \times 16$ (iii) $125 \times 40 \times 8 \times 25$.
6. Find the value of the following:
   (i) $54279 \times 92 + 54279 \times 8$ (ii) $60678 \times 262 - 60678 \times 162$
7. Find the following products by using suitable properties:
   (i) 739×102 (ii) 1938×99 (iii) 1005×188.
8. Divide 7750 by 17 and check the result by division algorithm.
9. Find the number which when divided by 38 gives the quotient 23 and remainder 17.
10. Which least number should be subtracted from 1000 so that the difference is exactly divisible by 35?
11. Which least number should be added to 1000 so that 53 divides the sum exactly?
12. Find the largest three-digit number which is exactly divisible by 47.
13. Find the smallest five-digit number which is exactly divisible by 254.
14. A vendor supplies 72 litres of milk to a student's hostel in the morning and 28 litres of milk in the evening every day. If the milk costs ₹ 39 per litre, how much money is due to the vendor per day?
15. State whether the following statements are true (T) or false (F):
   (i) If the product of two whole numbers is zero, then atleast one of them will be zero.
   (ii) If the product of two whole numbers is 1, then each of them must be equal to 1.
   (iii) If a and b are whole numbers such that $a \neq 0$ and $b \neq 0$, then ab may be zero.
16. Replace each * by the correct digit in each of the following:

(i)
```
   3 5 6
 - * 6 *
 -------
     * 9
```
(ii)
```
   6 5 0 *
 - * 0 * 5
 ---------
   4 * 5 7
```
(iii)
```
   1 7 0 0 * 4
 -   8 * * 4 7
 -------------
     * 8 6 6 *
```

PATTERNS IN WHOLE NUMBERS

Patterns in numbers are not only interesting, they help us in simplifying some calculations.

Let us observe the following patterns:

1. $5328 + 9 = 5328 + 10 - 1 = 5338 - 1 = 5337$
   $5328 + 99 = 5328 + 100 - 1 = 5428 - 1 = 5427$
   $5328 + 999 = 5328 + 1000 - 1 = 6328 - 1 = 6327$
   ....
   ....

This pattern helps in adding the numbers of the form 9, 99, 999, 9999, ...

2. $5328 - 9 = 5328 - 10 + 1 = 5318 + 1 = 5319$

$5328 - 99 = 5328 - 100 + 1 = 5228 + 1 = 5229$

$5328 - 999 = 5328 - 1000 + 1 = 4328 + 1 = 4329$

This pattern helps in subtracting the numbers of the form 9, 99, 999, ...

3. $257 \times 9 = 257 \times (10 - 1) = 2570 - 257 = 2313$

$257 \times 99 = 257 \times (100 - 1) = 25700 - 257 = 25443$

$257 \times 999 = 257 \times (1000 - 1) = 257000 - 257 = 256743$

This pattern helps in multiplying by numbers of the form 9, 99, 999, ...

4. $1 + 3 = 4 = 2 \times 2$

$1 + 3 + 5 = 9 = 3 \times 3$

$1 + 3 + 5 + 7 = 16 = 4 \times 4$

Can you write the next two steps of this pattern?

5. $1 \times 8 + 1 = 9$

$12 \times 8 + 2 = 98$

$123 \times 8 + 3 = 987$

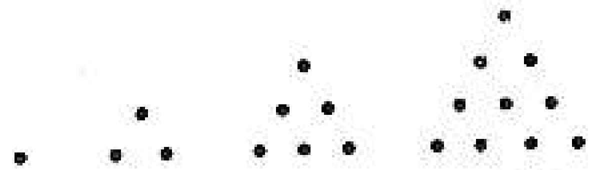
$1234 \times 8 + 4 = 9876$

Can you write the next two steps of this pattern?

Arrangement of numbers in elementary shapes

Some numbers can be arranged in elementary shapes made up of dots. Let one dot '•' represent number 1.

1. Look at the following figures made up of dots:

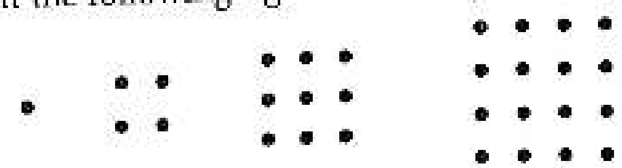


These figures show the arrangements of the numbers 1, 3, 6 and 10 by triangles. The numbers 1, 3, 6, 10, ... are called **triangular numbers.**

The next triangular number = 10 + 5 = 15.

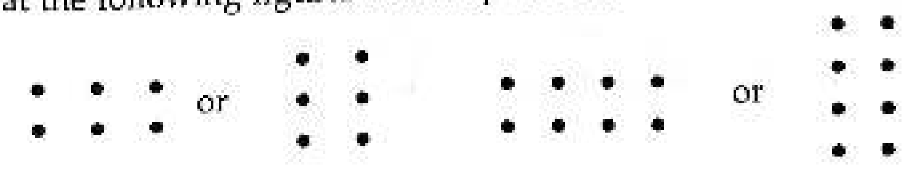
Can you write the next two triangular numbers?

2. Look at the following figures made up of dots:



These figures show the arrangement of the numbers 1, 4, 9 and 16. The numbers 1, 4, 9, 16, ... are called **square numbers.** Can you write the next two square numbers?

3. Look at the following figures made up of dots:



These figures show the arrangement of the numbers 2×3 or 3×2; 2×4 or 4×2 *i.e.* the numbers 6 and 8.

The numbers 6 and 8 are called **rectangular numbers**. Can you write two more rectangular numbers?

Exercise 2.3

1. Using shorter method, find
   (*i*) $3246 + 9999$ (*ii*) $7501 + 99999$ (*iii*) $5377 - 999$
   (*iv*) $25718 - 9999$ (*v*) 123×999 (*vi*) 203×9999
2. Without using a diagram, find
   (*i*) 9th square number (*ii*) 7th triangular number
3. (*i*) Can a rectangular number be a square number?
   (*ii*) Can a triangular number be a square number?
4. Observe the following pattern and fill in the blanks:
   $1 \times 9 + 1 = 10$
   $12 \times 9 + 2 = 110$
   $123 \times 9 + 3 = 1110$
   $1234 \times 9 + 4 =$ ....
   $12345 \times 9 + 5 =$ ....
5. Observe the following pattern and fill in the blanks:
   $9 \times 9 + 7 = 88$
   $98 \times 9 + 6 = 888$
   $987 \times 9 + 5 = 8888$
   $9876 \times 9 + 4 =$ ....
   $98765 \times 9 + 3 =$ ....

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (*i*) A whole number is less than all those whole numbers that lie to its .... on the number line.
   (*ii*) One more than a given whole is called its ....
   (*iii*) There is atleast one whole number between two .... whole numbers.
   (*iv*) $738 \times 335 = 738 \times (300 + 30 + ...)$
   (*v*) If a is a non-zero whole number and $a \times a = a$, then $a =$ ...
   (*vi*) .... is the only whole number which is not a natural number.
   (*vii*) The additive identity in whole numbers is ...

2. State whether the following statements are true (T) or false (F):
   (i) The predecessor of a 3-digit number is always a 3-digit number.
   (ii) The successor of a 3-digit number is always a 3-digit number.
   (iii) If a is any whole number, then $a \div a = 1$.
   (iv) If a is any non-zero whole number, then $0 \div a = 0$.
   (v) On adding two different whole numbers, we always get a natural number.
   (vi) Between two whole numbers there is a whole number.
   (vii) There is a natural number which when added to a natural number, gives that number.
   (viii) If the product of two whole numbers is zero, then atleast one of them is zero.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 16):

3. The whole number which does not have a predecessor in whole number system is
   (a) 0 (b) 1 (c) 2 (d) none of these
4. The predecessor of the smallest 4-digit number is
   (a) 99 (b) 999 (c) 1000 (d) 1001
5. The predecessor of 1 million is
   (a) 9999 (b) 99999 (c) 999999 (d) 1000001
6. The product of the predecessor and the successor of the greatest 2-digit number is
   (a) 9900 (b) 9800 (c) 9700 (d) none of these
7. The sum of the successor of the greatest 3-digit number and the predecessor of the smallest 3-digit number is
   (a) 1000 (b) 1100 (c) 1101 (d) 1099
8. The number of whole numbers between 22 and 54 is
   (a) 30 (b) 31 (c) 32 (d) 42
9. The number of whole numbers between the smallest whole number and the greatest 2-digit number is
   (a) 100 (b) 99 (c) 98 (d) 88
10. If a is whole number such that $a + a = a$, then a is equal to
   (a) 0 (b) 1 (c) 2 (d) none of these
11. The value of $(93 \times 63 + 93 \times 37)$ is
   (a) 930 (b) 9300 (c) 93000 (d) none of these
12. Which of the following is not equal to zero?
   (a) 0×5 (b) $0 \div 5$ (c) $(10 - 10) \div 5$ (d) $(5 - 0) \div 5$
13. Which of the following statement is true?
   (a) $21 - (13 - 5) = (21 - 13) - 5$
   (b) $21 - 13$ is not a whole number
   (c) $21 \times 1 = 21 \times 0$
   (d) $13 - 21$ is not a whole number
14. Which of the following statement is not true?
   (a) Zero is the identity for multiplication of whole numbers.
   (b) Addition and multiplication both are commutative for whole numbers.
   (c) Addition and multiplication both are associative for whole numbers.
   (d) Multiplication is distributive over addition for whole numbers.

15. On dividing a number by 9 we get 47 as quotient and 5 as remainder. The number is
 (*a*) 418 (*b*) 428 (*c*) 429 (*d*) none of these
16. By using dot (•) pattern, which of the following numbers can be arranged in two ways namely a triangle and a rectangle?
 (*a*) 12 (*b*) 11 (*c*) 10 (*d*) 9

Higher Order Thinking Skills (HOTS)

1. The height of a slippery pole is 10 m and an insect is trying to climb the pole. The insect climbs 5 m in one minute and then slips down by 4 m. In how much time will insect reach the top?
2. Which is greater, the sum of first twenty whole numbers or the product of first twenty whole numbers?
3. If a whole number is divisible by 2 and 4, is it divisible by 8 also?

Summary

★ All natural numbers together with 0 are called whole numbers.
★ Every natural number is a whole number.
★ 0 is a whole number but not a natural number.
★ 0 is the smallest whole number.
★ There is no largest whole number.
★ The successor of a whole number is one more than the given number.
★ The predecessor of a whole number (except zero) is one less than the given number.
★ Of the two given different whole numbers, the one which lies to the right on the number line is greater than the other.
★ There is no whole number between two consecutive whole numbers.
★ There is atleast one whole number between two non-consecutive whole numbers.
★ **Addition properties of whole numbers**
 ❑ *Closure property* — If a and b are any whole numbers then $a + b$ is also a whole number.
 ❑ *Commutative property* — If a and b are any whole numbers then $a + b = b + a$.
 ❑ *Associative law* — If a, b and c are any whole numbers then $(a + b) + c = a + (b + c)$.
 ❑ *Additive identity* — If a is any whole number then $a + 0 = a = 0 + a$.
 ❑ *Cancellation law* — If a, b and c are any whole numbers then $a + c = b + c \Rightarrow a = b$.
★ **Multiplication properties of whole numbers**
 ❑ *Closure property* — If a and b are any whole numbers then $a \times b$ is also a whole number.
 ❑ *Commutative property* — If a and b are any whole numbers then $a \times b = b \times a$.
 ❑ *Associative law* — If a, b and c are any whole numbers then $(a \times b) \times c = a \times (b \times c)$.
 ❑ *Distributive law* — If a, b and c are any whole numbers then $a \times (b + c) = a \times b + a \times c$.
 ❑ *Multiplicative identity* — If a is any whole number, then $a \times 1 = a = 1 \times a$.
 ❑ *Multiplication by zero* — If a is any whole number, then $a \times 0 = 0 = 0 \times a$.

- *Cancellation law* — If a and b are any whole numbers and c is a non-zero whole number then $a \times c = b \times c \Rightarrow a = b$.

★ If a, b ($\neq 0$) and c are whole numbers such that $b \times c = a$ then $a \div b = c$.

★ Division by zero is not defined.

★ **Properties of division of whole numbers**

- If a is any whole number, then $a \div 1 = a$.
- If a is any non-zero whole number, then $a \div a = 1$.
- If a is any non-zero whole number, then $0 \div a = 0$.

★ **Division algorithm**

If a is any whole number and b is another smaller non-zero whole number then there exist unique whole numbers q and r such that $a = b \times q + r$ where $0 \leq r < b$.

Check Your Progress

1. Write next three consecutive whole numbers of the number 9998.
2. Write three consecutive whole numbers occurring just before 567890.
3. Find the product of the successor and the predecessor of the smallest number of 3-digits.
4. Find the number of whole numbers between the smallest and the greatest numbers of 2-digits.
5. Find the following sum by suitable arrangements:
   (*i*) 678 + 1319 + 322 + 5681 (*ii*) 777 + 546 + 1463 + 223 + 537
6. Determine the following products by suitable arrangements:
   (*i*) $625 \times 437 \times 16$ (*ii*) $309 \times 25 \times 7 \times 8$
7. Find the value of the following by using suitable properties:
   (*i*) $236 \times 414 + 236 \times 563 + 236 \times 23$ (*ii*) $370 \times 1587 - 37 \times 10 \times 587$
8. Divide 6528 by 29 and check the result by division algorithm.
9. Find the greatest 4-digit number which is exactly divisible by 357.
10. Find the smallest 5-digit number which is exactly divisible by 279.

3 INTEGERS

INTRODUCTION

Ruchi's mother has 7 oranges. Ruchi has to go for a picnic with her friends. She wants to carry 10 oranges. Can her mother give 10 oranges to her? She does not have enough, so she borrows 3 oranges from her neighbour to be returned later. After giving 10 oranges to Ruchi, how many oranges are left with her mother? Can we say that she has zero oranges? She has no oranges with her but has to return 3 oranges to her neighbour. So when she gets some more oranges from the market, say 8, she will return 3 oranges and will be left with 5 oranges only.

Antony goes to the market to purchase a notebook. He has ₹ 20 with him but the notebook costs ₹ 26. The shopkeeper write ₹ 6 as due amount from him. He writes ₹ 6 in his diary to remember Antony's debt. But how would he remember whether ₹ 6 has to be given or to be taken from Antony? Can he express his debt by some colour or sign?

In this chapter, you will learn:

- Integers
- Representation of integers on number line
- Absolute value of an integer
- Comparison of integers
- Uses of integers
- Addition of integers using number line
- Addition of integers
- Properties of addition of integers
- Subtraction of integers using number line
- Successor and predecessor of an integer
- Addition/subtraction of three or more integers.

INTEGERS

We know that on adding any two whole numbers, we always get a whole number. Is this true for subtraction as well? Let us examine it.

Consider the following examples:

$37 - 25 = 12$

$37 - 37 = 0$

$25 - 37 = ?$

We observe that in the last case, there is no answer in the system of whole numbers *i.e.* when a bigger whole number is subtracted from a smaller whole number, we do not get a whole number; thus the system of whole numbers is inadequate for subtraction. Therefore, there is a need to enlarge the system of whole numbers to provide an answer to all questions of subtraction. We introduce negative numbers $-1, -2, -3, -4, -5, \ldots$ which are opposite to natural numbers $1, 2, 3, 4, 5, \ldots$.

These new numbers together with whole numbers are called **integers.**

Thus, the numbers ....., $-4, -3, -2, -1, 0, 1, 2, 3, 4$, ... are **integers.**

The natural numbers 1, 2, 3, 4, ... are called **positive integers.**

The negative numbers $-1, -2, -3, -4$, ... are called **negative integers.**

The number 0 is an integer. It is neither positive nor negative.

Positive integers are also written as +1, +2, +3, ...

However, the plus sign (+) is usually omitted.

Representation of integers on a number line

To represent integers on a line, we proceed as under:

(1) Draw a straight line, mark a point on it and label it as 0 (zero).

(2) Select a unit of length. Mark points on the line at unit length intervals from each other on both sides of 0 (zero).

(3) Label the points to the right of 0 (zero) successively as 1, 2, 3, ... and the points to the left of 0 (zero) successively as $-1, -2, -3, \ldots$

(4) Put arrows on each side of the line to show that the line continues indefinitely in both directions.

Thus, every integer can be represented by some point on the line. The line drawn above is called the **number line.**

From the number line, we observe that:

1. *There is no largest integer and no smallest integer.*
2. *An integer is greater than all those integers that lie to its left on the number line.*

In other words, an integer is greater than the other integer if the first integer lies on the right of the second integer on the number line.

For example:

As 5 lies on the right of -2 on the number line, so $5 > -2$.

Similarly, $0 > -3$ and $-1 > -4$.

3. *An integer is less than all those integers that lie to its right on the number line.*

In other words, an integer is less than the other integer if the first integer lies on the left of the second integer on the number line.

For example:

As -2 lies on the left of 5 on the number line, so $-2 < 5$.

Similarly, $-3 < 0$ and $-4 < -1$.

From the above observations it follows that:

(i) *Zero is less than every positive integer.*

(ii) *Zero is greater than every negative integer.*

(iii) *Every positive integer is greater than every negative integer.*

(iv) *Farther a number from zero on the right, larger is its value.*

(v) *Farther a number from zero on the left, smaller is its value.*

Absolute value of an integer

The **absolute value** of an integer is its numerical value regardless of its sign.

The absolute value of an integer a is written as $|a|$.

Thus, $|5| = 5$, $|13| = 13$, $|0| = 0$, $|-3| = 3$, $|-7| = 7$ etc.

Note that the absolute value of an integer is always non-negative.

If a is an integer, then

(i) $|a| = a$ if $a \geq 0$ *i.e.* if a is positive or zero.

(ii) $|a| = -a$ if $a < 0$ *i.e.* if a is negative.

Comparison of integers

The rules for the comparison of integers are:

1. Every positive integer is greater than zero and every negative integer is less than zero.
2. Every positive integer is greater than every negative integer; *alternatively*, every negative integer is less than every positive integer.
3. Given two positive integers – compare them as whole numbers.
4. Given two negative integers – the integer with smaller absolute value is greater; *alternatively*, the integer with bigger absolute value is smaller.

For example:

(i) $-3 > -7$ because $|-3| < |-7|$ *i.e.* $3 < 7$.

(ii) $-25 > -39$ because $|-25| < |-39|$ *i.e.* $25 < 39$.

(iii) $-78 < -43$ because $|-78| > |-43|$ *i.e.* $78 > 43$.

Uses of integers

In our daily life, we come across many statements which are opposite to each other. Integers are used to express these statements in mathematical terms.

For example:

(i) Profits are represented by positive integers and losses are represented by negative integers.

(ii) Deposits in a bank (or post office etc.) are represented by positive integers and withdrawals are represented by negative integers.

(iii) Heights above sea level are represented by positive integers and depths below sea level are represented by negative integers.

(iv) Temperatures above freezing point are represented by positive integers and below freezing point are represented by negative integers and so on.

Thus, a gain of ₹500 is represented by + ₹500 (or ₹500) and a loss of ₹500 is represented by – ₹500.

Example 1. *Write opposite of the following:*

(*i*) *Decrease in weight* (*ii*) *Profit of ₹7500*

(*iii*) *Walking 800 m towards north* (*iv*) *70 m below sea level.*

Solution. (*i*) Increase in weight (*ii*) Loss of ₹7500

(*iii*) Walking 800 m towards south (*iv*) 70 m above sea level.

Example 2. *Write each of the following using appropriate sign '+' or '–':*

(*i*) *A withdrawal of ₹2000.*

(*ii*) *An aeroplane is flying at a height of 3500 m above sea level.*

(*iii*) *5°C below 0°C.*

Solution. (*i*) –₹2000 (*ii*) +3500 m (*iii*) –5°C.

Example 3. *Draw a number line and answer the following questions:*

(*i*) *Which integers lie between –6 and –1?*

(*ii*) *Which is the largest integer among them?*

(*iii*) *Which is the smallest integer among them?*

Solution. The number line is shown below:

(*i*) The integers between –6 and –1 are –5, –4, –3, –2.

(*ii*) The largest integer among these is –2.

(*iii*) The smallest integer among these is –5.

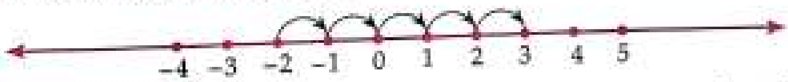
Example 4. *Draw a number line and answer the following questions:*

(*i*) *Which number will we reach if we move 5 units to the right of –2?*

(*ii*) *Which number will we reach if we move 6 units to the left of 1?*

(*iii*) *In which direction should we move to reach –7 from –2?*

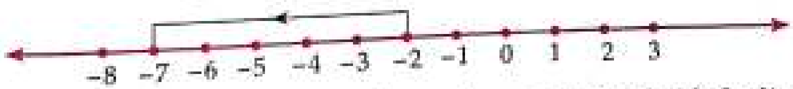
Solution. Draw the number line.

(*i*) 

After moving 5 units to the right of –2, we reach at 3.

(*ii*) 

After moving 6 units to the left of 1, we reach at –5.

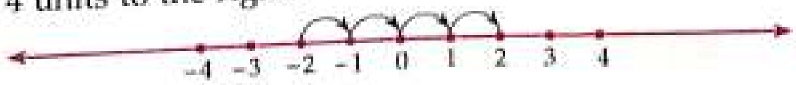
(*iii*) 

To reach –7 from –2, we have to move in the left direction.

Example 5. *Using the number line, write the integer which is:*

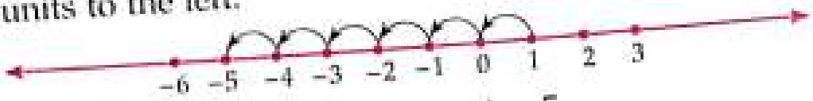
(*i*) *4 more than –2* (*ii*) *6 less than 1* (*iii*) *5 less than –1.*

Solution. (*i*) Since we want to find an integer which is 4 more than –2, so we start from –2 and move 4 units to the right.



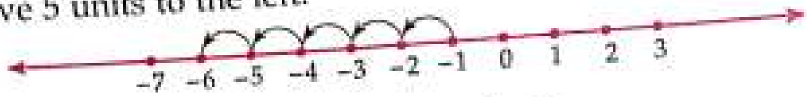
We reach at 2, so the required integer is 2.

(*ii*) Since we want to find an integer which is 6 less than 1, so we start from 1 and move 6 units to the left.



We reach at –5, so the required integer is –5.

(*iii*) Since we want to find an integer which is 5 less than –1, so we start from –1 and move 5 units to the left.



We reach at –6, so the required integer is –6.

Example 6. *Evaluate the following:*

(*i*) $|-7| + |10|$ (*ii*) $|-12| + |-9|$ (*iii*) $|-8| - |6|$ (*iv*) $|-15| - |-11|$.

Solution.

(*i*) Since $|-7| = 7$ and $|10| = 10$,
$\therefore |-7| + |10| = 7 + 10 = 17$.

(*ii*) Since $|-12| = 12$ and $|-9| = 9$,
$\therefore |-12| + |-9| = 12 + 9 = 21$.

(*iii*) Since $|-8| = 8$ and $|6| = 6$,
$\therefore |-8| - |6| = 8 - 6 = 2$.

(*iv*) Since $|-15| = 15$ and $|-11| = 11$
$\therefore |-15| - |-11| = 15 - 11 = 4$.

Example 7. *Use the appropriate symbol < or > to fill in the following blanks:*

(*i*) –5 ... 3 (*ii*) 5 ... –1 (*iii*) 0 ... –6
(*iv*) –10 ... 10 (*v*) –5 ... –3 (*vi*) –64 ... –203.

Solution.

(*i*) As every negative integer is less than every positive integer, $-5 < 3$.

(*ii*) As every positive integer is greater than every negative integer, $5 > -1$.

(*iii*) As zero is greater than every negative integer, $0 > -6$.

(*iv*) As every negative integer is less than positive integer, $-10 < 10$.

(*v*) As $|-5| > |-3|$ *i.e.* $5 > 3$, $-5 < -3$.

(*vi*) As $|-64| < |-203|$ *i.e.* $64 < 203$, $-64 > -203$.

Example 8. *Arrange the following integers in ascending order:*

–33, 37, 5, 615, –9.

Solution. For negative integers –33, –9,

we have $-33 < -9$ because $|-33| > |-9|$ *i.e.* $33 > 9$.

For positive integers 37, 5, 615, we have $5 < 37 < 615$.

As every negative integer is less than every positive integer, therefore, we get
$-33 < -9 < 5 < 37 < 615$.

Hence, the given integers in ascending order are:
–33, –9, 5, 37, 615.

Ascending means smaller to greater

Example 9. *Arrange the following integers in descending order:*

189, 2056, −49, −6, −11, 678.

Solution. For positive integers 189, 2056, 678,

we have 2056 > 678 > 189.

For negative integers −49, −6, −11,

we have −6 > −11 > −49

because |−6| < |−11| < |−49| *i.e.* 6 < 11 < 49.

As every positive integer is greater than every negative integer, therefore, we get

2056 > 678 > 189 > −6 > −11 > −49.

Hence, the given integers in descending order are:

2056, 678, 189, −6, −11, −49.

Descending means greater to smaller

Exercise 3.1

1. Write the opposite of the following:
   (*i*) Loss of ₹5000
   (*ii*) 30 km East of Delhi
   (*iii*) 200 m above sea level
   (*iv*) 325 BC
   (*v*) Spending ₹2700
   (*vi*) 25°C above freezing point.
2. Write each of the following using appropriate sign '+' or '−':
   (*i*) Gain of 3 kg weight
   (*ii*) Earning ₹1340
   (*iii*) 20°C below freezing point
   (*iv*) Loss of ₹470
   (*v*) Depositing ₹2500 in a bank
   (*vi*) 240 m below sea level
   (*vii*) A jet plane flying at a height of 9320 m.
   (*viii*) 6 m down in the basement of a building.
3. In each of the following pairs, which number is to the right of the other on the number line?
   (*i*) 3, 5 (*ii*) 0, −2 (*iii*) −3, −5 (*iv*) 2, −7.
4. In each of the following pairs, which number is to the left of the other on the number line?
   (*i*) −3, 0 (*ii*) 2, −5 (*iii*) −4, −7 (*iv*) −10, −16.
5. Draw a number line and answer the following questions:
   (*i*) Which integers lie between −9 and −2?
   (*ii*) Which is the largest among them?
   (*iii*) Which is the smallest among them?
6. Write four consecutive integers just greater than −9.
7. Write four consecutive integers just before −2.
8. Draw a number line and answer the following questions:
   (*i*) Which number will we reach if we move 6 units to the right of −1?
   (*ii*) Which number will we reach if we move 7 units to the left of 2?
   (*iii*) In which direction should we move to reach 3 from −3?
   (*iv*) In which direction should we move to reach −8 from −3?

9. Using the number line, write the integer which is:
   (i) 5 more than −1
   (ii) 5 less than −1
   (iii) 7 less than 2
   (iv) 3 more than −7.

10. Evaluate the following:
   (i) $|13 - 5|$
   (ii) $|5 - 13|$
   (iii) $|-11| + |9|$
   (iv) $|-8| + |-6|$
   (v) $|7| - |-3|$
   (vi) $|-19| - |-13|$.

11. Use the appropriate symbol < or > to fill in the following blanks:
   (i) −3 7
   (ii) 0 ...... −2
   (iii) −10 ...... −11
   (iv) −6 ...... −2
   (v) −5 ...... −13
   (vi) −30 ...... −19.

12. Arrange the following integers in ascending order:
   (i) −5, 3, 0, −9, 2
   (ii) −28, −33, 9, −4, −31, −2, 35

13. Arrange the following integers in descending order:
   (i) −31, 25, −37, 43, 0, −5
   (ii) −101, 95, −3, −8, 36, −7, −84.

14. State whether the following statements are true (T) or false (F):
   (i) 0 is the smallest positive integer.
   (ii) Every negative integer is less than every natural number.
   (iii) −7 is to the right of −6 on the number line.
   (iv) The absolute value of an integer is always greater than the integer.

ADDITION OF INTEGERS

Addition of integers using number line

To add a positive integer, move to the right on the number line.

Example 1. *Find* (−5) + 3.

Solution. Start from −5 on the number line.

Move 3 units to the right. We reach at −2.

∴ (−5) + 3 = −2.

To add a negative integer, move to the left on the number line.

Example 2. *Find* 5 + (−3).

Solution. Start from 5 on the number line.

Move 3 units to the left. We reach at 2.

∴ 5 + (−3) = 2.

Example 3. *Find* (−3) + (−2).

Solution. Start from −3 on the number line.

Move 2 units to the left. We reach at −5.

∴ (−3) + (−2) = −5.

Addition of integers

Using number line to add integers which have large absolute values is very inconvenient.

As a better alternative, we have the following rules for addition of integers:

- To add two positive integers—add them as natural numbers.
- To add two negative integers—add their absolute values and give the negative sign to the sum obtained.

  In practice, ignore their signs, find the sum of the numbers and give negative sign to the sum obtained.
- To add a positive integer and a negative integer – subtract the smaller absolute value from the larger absolute value and give the sign of the integer which has the larger absolute value to the result obtained.

  In practice, ignore their signs, subtract the smaller number from the larger number and give the sign of the integer which has the larger absolute value to the result obtained.

Example 4. *Evaluate the following:*

(*i*) $(+129) + (+274)$ (*ii*) $(-78) + (-125)$.

Solution. (*i*) As both integers are positive, so add them like natural numbers.

$\therefore$ $(+129) + (+274) = 129 + 274 = 403$.

(*ii*) As both integers are negative, so add their absolute values and give the negative sign to the sum obtained.

Now $|-78| = 78$ and $|-125| = 125$.

Adding their absolute values, we get $78 + 125 = 203$.

$\therefore$ $(-78) + (-125) = -203$.

In practice, we write the solution as under:

$(-78) + (-125) = -(78 + 125) = -203$.

Example 5. *Evaluate the following:*

(*i*) $(-48) + 85$ (*ii*) $136 + (-234)$.

Solution. As the two integers have different signs, so subtract the smaller absolute value from the larger absolute value and give the sign of the integer which has larger absolute value.

(*i*) Here, $|-48| = 48$ and $|85| = 85$.

Subtract 48 from 85

As the sign of the integer with larger absolute value is positive, so put positive sign before the result.

$\therefore$ $(-48) + 85 = +(85 - 48) = +37 = 37$.

In practice, we write the solution as under:

$(-48) + 85 = +(85 - 48) = +37 = 37$.

(*ii*) Here, $|136| = 136$ and $|-234| = 234$.

Subtract 136 from 234.

As the sign of the integer with larger absolute value is negative, so put negative sign before the result.

∴ $136 + (-234) = -(234 - 136) = -98$.

In practice, we write the solution as under:

$136 + (-234) = -(234 - 136) = -98$.

Example 6. *Evaluate the following:*

(*i*) $-2051 + (-759)$ (*ii*) $859 + (-2737)$ (*iii*) $-2057 + 4718$

Solution. (*i*) Add 2051 to 759 and put negative sign before the result.

∴ $-2051 + (-759) = -(2051 + 759) = -2810$.

(*ii*) Subtract 859 from 2737 and put negative sign before the result.

∴ $859 + (-2737) = -(2737 - 859) = -1878$.

(*iii*) Subtract 2057 from 4718 and put positive sign before the result.

∴ $-2057 + 4718 = +(4718 - 2057) = +2661 = 2661$.

Properties of addition of integers

- **Closure property of addition**

Let us add any two integers and check whether the sum is an integer.

Integer	*Integer*	*Sum*	*Is the sum an integer?*
4	7	$4 + 7 = 11$	Yes
–5	–8	$(-5) + (-8) = -13$	Yes
–6	9	$(-6) + 9 = 3$	Yes
–11	3	$(-11) + 3 = -8$	Yes

Thus, we find that the sum of two integers is an integer. In other words:
If a and b are any two integers then a + b is also an integer. This is called *closure property of addition*.

- **Commutative property of addition**

If a and b are any two integers, then $a + b = b + a$.

For example:

(*i*) $4 + 7 = 11$ and $7 + 4 = 11$

∴ $4 + 7 = 7 + 4$.

(*ii*) $(-5) + 14 = 9$ and $14 + (-5) = 9$

∴ $(-5) + 14 = 14 + (-5)$.

- **Associative law of addition**

If a, b and c are any integers, then $(a + b) + c = a + (b + c)$.

For example:

(*i*) $(4 + 7) + 13 = 11 + 13 = 24$ and $4 + (7 + 13) = 4 + 20 = 24$

∴ $(4 + 7) + 13 = 4 + (7 + 13)$.

(*ii*) $[(-5) + 7] + (-6) = 2 + (-6) = -4$ and $(-5) + [7 + (-6)] = (-5) + 1 = -4$

∴ $[(-5) + 7] + (-6) = (-5) + [7 + (-6)]$.

In view of the above results, we can drop brackets and we write

$$(a + b) + c = a + (b + c) = a + b + c.$$

- **Existence of additive identity**

For every integer a, $a + 0 = a = 0 + a$.

For example:

$$(-21) + 0 = -21 = 0 + (-21).$$

The integer 0 is called the additive identity.

- **Additive inverse**

For every integer a, there exists integer $-a$ such that $a + (-a) = 0 = (-a) + a$.

For example:

$$5 + (-5) = 0 = (-5) + 5.$$

Thus, $-a$ is the additive inverse a and a is the additive inverse of $-a$.

Hence, $-(-a) = a$.

Exercise 3.2

1. Evaluate the following, using the number line:
   (*i*) $4 + (-5)$ (*ii*) $(-4) + 5$ (*iii*) $7 + (-3)$ (*iv*) $-6 + (-2)$.
2. Evaluate the following:
   (*i*) $(-8) + (-14)$ (*ii*) $-35 + (-47)$ (*iii*) $91 + (-48)$
   (*iv*) $(-203) + 501$ (*v*) $(-36) + 29$ (*vi*) $(-131) + 97$.
3. Evaluate the following:
   (*i*) $-1083 + (-3974)$ (*ii*) $706 + (-394)$ (*iii*) $1309 + (-2811)$.
4. Fill in the following blanks:
   (*i*) $-(-5) = \cdots$ (*ii*) $-(-30) = \cdots$ (*iii*) $-(-539) = \cdots$
5. Write down the additive inverses of:
   (*i*) 9 (*ii*) -11 (*iii*) -237 (*iv*) 567.

SUBTRACTION OF INTEGERS

Subtraction of integers using a number line

In addition we combine two numbers, while in subtraction we take away one number from another. Thus, ***subtraction is the opposite of addition.***

Recall that, to add a positive integer you move to the right, so:

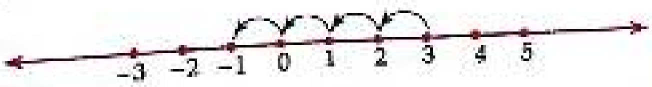
> ***To subtract a positive integer, move to the left on the number line.***

Example 1. *Find 3 – 4.*

Solution. Start from 3 on the number line.

Move 4 units to the left. We reach at -1.

$\therefore \quad 3 - 4 = -1.$



Example 2. *Find* (–3) –4.

Solution. Start from –3 on the number line.

Move 4 units to the left. We reach at –7.

∴ (–3) –4 = –7.

Recall that, to add a negative integer you move to the left, so:

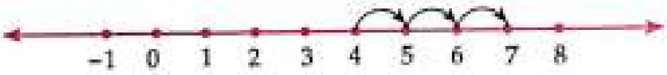
> ***To subtract a negative integer, move to the right on the number line.***

Example 3. *Find* 4 – (–3).

Solution. Start from 4 on the number line.

Move 3 units to the right. We reach at 7.

∴ 4 – (–3) = 7.



Example 4. *Find* (–2) – (–5).

Solution. Start from –2 on the number line.

Move 5 units to the right. We reach at 3.

∴ –2 – (–5) = 3.

Subtraction of integers

Subtracting one integer from another integer is same as adding the additive inverse (opposite) of the integer that is being subtracted to the other integer.

For example:

(*i*) Subtracting 7 from 12 is same as adding –7 (opposite of 7) to 12,
∴ 12 – 7 = 12 + (–7) = 5.

(*ii*) Subtracting 5 from –11 is same as adding –5 (opposite of 5) to –11,
∴ –11 – 5 = –11 + (–5) = –16.

(*iii*) Subtracting –9 from 16 is same as adding 9 (opposite of –9) to 16,
∴ 16 – (–9) = 16 + 9 = 25.

(*iv*) Subtracting –5 from –13 is same as adding 5 (opposite of –5) to –13,
∴ –13 – (–5) = –13 + 5 = –8.

Thus, we have:

> ***If a and b are any two integers, then a – b = a + (–b).***

In other words, change the sign of the integer to be subtracted and then add.

Example 5. *Subtract:*

(*i*) *–7 from 12* (*ii*) *12 from –7* (*iii*) *–12 from –7* (*iv*) *–235 from –411.*

Solution. (*i*) $12 - (-7) = 12 + 7$

$= 19$

Change the sign of −7 and add

(*ii*) $-7 - 12 = (-7) + (-12)$

$= -(7 + 12) = -19$

Change the sign of 12 and add

(*iii*) $-7 - (-12) = -7 + 12 = +(12 - 7) = +5 = 5$

(*iv*) $-411 - (-235) = -411 + 235 = -(411 - 235) = -176.$

Example 6. *Evaluate the following:*

(*i*) $(-526) - (-217)$ (*ii*) $(-239) - (+1573)$.

Solution. (*i*) $(-526) - (-217) = -526 + 217$

$= -(526 - 217) = -309$

Change the sign of −217 and add

(*ii*) $(-239) - (+1573) = (-239) + (-1573)$

$= -(239 + 1573) = -1812.$

Change the sign of + 1573 and add

Example 7. *The sum of two integers is −23. If one of them is −5, find the other.*

Solution. Other integer = sum of two integers − (the given integer)

$= -23 - (-5) = -23 + 5$

$= -(23 - 5) = -18.$

Successor and predecessor of an integer

One more than a given integer is called its ***successor****.*

One less than a given integer is called its ***predecessor****.*

Thus, if a is an integer then

(*i*) its successor is $a + 1$. (*ii*) its predecessor is $a - 1$.

Example 8. *Find the successor and the predecessor of −23.*

Solution. Successor of $-23 = -23 + 1 = -22$ and

predecessor of $-23 = -23 - 1 = -24$.

Exercise 3.3

1. Evaluate the following, using the number line:

   (*i*) $4 - (-2)$ (*ii*) $-4 - (-2)$ (*iii*) $3 - 6$ (*iv*) $-3 - (-5)$.

2. Subtract:

   (*i*) −6 from 9 (*ii*) 6 from −9 (*iii*) −6 from −9

   (*iv*) −725 from −63 (*v*) −376 from 10 (*vi*) 92 from −620.

3. Evaluate the following:

   (*i*) $-237 - (+1884)$ (*ii*) $-346 - (-1275)$

   (*iii*) $-190 - (-3512)$ (*iv*) $-2718 - (+6827)$.

4. The sum of two integers is 17. If one of them is −35, find the other.

5. What must be added to −23 to get −9?

6. Find the predecessor of 0.
7. Find the successor and the predecessor of the following integers:
   (*i*) −31 (*ii*) −735 (*iii*) −240.

ADDITION/SUBTRACTION OF THREE OR MORE INTEGERS

Three or more integers can be added/subtracted by the successive application of the rules practised above. However, it is more convenient to group the positive and negative integers separately.

Example 1. *Find 9 + (−3) − (−2).*

Solution. *Step 1.* Add the first two numbers.

$9 + (-3) = 9 - 3 = 6$

Step 2. Add the result of step 1 to the third number.

$\therefore \quad 9 + (-3) - (-2) = 6 - (-2) = 6 + 2 = 8.$

Example 2. *Evaluate the following:*

(*i*) $280 + (-130) - 96$ (*ii*) $372 + (-584) - (-98)$ (*iii*) $-146 + (-78) - (-124) + 69.$

Solution. (*i*) $280 + (-130) - 96 = 280 - 130 - 96$

$= 280 - (130 + 96)$ — Add negative integers together

$= 280 - 226 = 54$

(*ii*) $372 + (-584) - (-98) = 372 - 584 + 98$

$= (372 + 98) - 584$ — Add positive integers together

$= 470 - 584$

$= -(584 - 470) = -114$

(*iii*) $-146 + (-78) - (-124) + 69$

$= -146 - 78 + 124 + 69$

$= -(146 + 78) + (124 + 69)$ — Add positive integers separately and negative integers separately

$= -224 + 193$

$= -(224 - 193) = -31.$

Example 3. *Find 7 − 5 + 4 + 3 − 2 − 6 − 8.*

Solution. *Step 1.* Group the positive and negative integers separately.

	−5
+7	−2
+4	−6
+3	−8
+14	−21

Step 2. Add the two results of step 1.

$(+14) + (-21) = 14 - 21 = -7.$

Exercise 3.4

1. Find the value of:
   (*i*) 6 − 9 + 4 (*ii*) −5 − (−3) + 2 (*iii*) 7 + (−5) + (−6) (*iv*) 6 − 3 − (−5).
2. Evaluate the following:
   (*i*) −77 + (−84) + 318 (*ii*) 54 + (−218) − (−76)
   (*iii*) −121 − (−78) + (−193) + 576 (*iv*) −65 + (−76) − (−28) + 32.
3. Find the value of:
   (*i*) 8 − 6 + (−2) − (−3) + 1 (*ii*) 31 + (−23) − 35 + 18 − 4 − (−3).
4. Rashmi deposited ₹4370 in her account on Monday and then withdrew ₹2875 on Tuesday. Next day she deposited ₹1550. What was her balance on Thursday?

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (*i*) The absolute value of 0 is .....
   (*ii*) The sum of two negative integers is always a ..... integer.
   (*iii*) The smallest positive integer is .....
   (*iv*) The largest negative integer is .....
   (*v*) 17 + ... = 0
   (*vi*) ..... −15 = −10
   (*vii*) The predecessor of −99 is .....
2. State whether the following statements are true (T) or false (F):
   (*i*) The sum of a positive integer and a negative integer is always a negative integer.
   (*ii*) Zero is an integer.
   (*iii*) The sum of an integer and its negative is always zero.
   (*iv*) The sum of three integers can never be zero.
   (*v*) |−7| < |−3|.
   (*vi*) −20 is to the left of −21 on the number line.
   (*vii*) The successor of −29 is −30.
   (*viii*) 0 is greater than every negative integer.
   (*ix*) The difference of two integers is always an integer.
   (*x*) Additive inverse of a negative integer is always a positive integer.
3. State whether the following statements are true or false. If a statement is false, write the corresponding correct statement.
   (*i*) −8 is to the right of −10 on the number line.
   (*ii*) −100 is to the right of −50 on the number line.
   (*iii*) Smallest negative integer is −1.
   (*iv*) −26 is greater than −25.
   (*v*) −187 is the predecessor of −188.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (4 to 17):

4. The integer which is 5 more than −2 is
   (*a*) −7 (*b*) −3 (*c*) 3 (*d*) 7
5. The number of integers between − 1 and 1 is
   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3
6. The number of integers between −3 and 2 are
   (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5
7. The number of whole numbers between − 6 and 6 is
   (*a*) 11 (*b*) 10 (*c*) 6 (*d*) 5
8. The greatest integer lying −10 and −15 is
   (*a*) −10 (*b*) −11 (*c*) −14 (*d*) −15
9. The smallest integer lying between −10 and −15 is
   (*a*) −10 (*b*) −11 (*c*) −14 (*d*) −15
10. Which of the following statement is true?
    (*a*) $|10 - 4| = |10| + |-4|$ (*b*) Additive inverse of −5 is 5
    (*c*) −1 lies on the right of 0 on the number line (*d*) −7 is greater than −3
11. Which of the following statement is false?
    (*a*) $-20 - (-5) = -15$ (*b*) $|-18| > |-13|$
    (*c*) $23 + (-31) = 8$ (*d*) Every negative integer is less than 5
12. Which of the following statements is false?
    (*a*) $(-3) + (-11)$ is an integer (*b*) $(-19) + 13 = 13 + (-19)$
    (*c*) $(-15) + 0 = -15 = 0 + (-15)$ (*d*) Negative of −7 does not exist
13. If the sum of two integers is −17 and one of them is −9, then the other is
    (*a*) 8 (*b*) −8 (*c*) 26 (*d*) −26
14. On subtracting −7 from −4, we get
    (*a*) 3 (*b*) −3 (*c*) −11 (*d*) none of these
15. $(-12) + 17 - (-10)$ is equal to
    (*a*) −5 (*b*) 5 (*c*) 15 (*d*) −15
16. Which of the following statements is true?
    (*a*) $-13 > -8 - (-6)$ (*b*) $-5 - 4 > -12 + 2$
    (*c*) $(-8) - 3 = (-3) - (-8)$ (*d*) $(-15) - (-22) < (-22) - (-15)$
17. The statement "when an integer is added to itself, the sum is less than the integer" is
    (*a*) always true (*b*) never true
    (*c*) true only when the integer is negative
    (*d*) true when the integer is zero or positive

Higher Order Thinking Skills (HOTS)

1. Can the sum of successor and predecessor of an integer be an odd integer?
2. What is the sum of all integers from −500 to 500?
3. Find two positive integers such that their product is 1,00,000 and none of them contains 0 as a digit.

Summary

- ★ The numbers ....., −4, −3, −2, −1, 0, 1, 2, 3, 4, ... are called integers.
- ★ The natural numbers 1, 2, 3, 4, ... are called positive integers.
- ★ The numbers −1, −2, −3, −4, ... are called negative integers.
- ★ The number 0 is an integer. It is neither positive nor negative.
- ★ Of the two given different integers, the one which lies to the right on the number line is greater than the other.
- ★ Of the two given different integers, the one which lies to the left on the number line is less than the other.
- ★ There is no largest integer and no smallest integer.
- ★ 1 is the smallest positive integer and −1 is the largest negative integer.
- ★ The absolute value of an integer a is its numerical value regardless of the sign of a. It is written as $|a|$.
- ★ **Comparison of integers**
  - ❑ Every positive integer is greater than zero and every negative integer is less than zero.
  - ❑ Every positive integer is greater than every negative integer, *alternatively*, every negative integer is less than every positive integer.
  - ❑ Given two positive integers – compare them as whole numbers.
  - ❑ Given two negative integers – the integer with smaller absolute value is greater; *alternatively*, the integer with greater absolute value is smaller.
- ★ **Addition/subtraction of integers using number line**
  - ❑ To add a positive integer, move to the right on the number line.
  - ❑ To add a negative integer, move to the left on the number line.
  - ❑ To subtract a positive integer, move to the left on the number line.
  - ❑ To subtract a negative integer, move to the right on the number line.
- ★ **Addition/subtraction of integers**
  - ❑ To add two positive integers — add them as natural numbers.
  - ❑ To add two negative integers — add their absolute values and give the negative sign to the sum obtained.
  - ❑ To add a positive integer and a negative integer — subtract the smaller absolute value from the larger absolute value and give the sign of the integer which has the larger absolute value to the result obtained.
  - ❑ To subtract an integer from another integer — change the sign of the integer to be subtracted and then add.
- ★ **Successor and predecessor of an integer**
  - ❑ If a is an integer, then its successor = $a + 1$ and its predecessor = $a - 1$.
  - ❑ Every integer has a successor as well as predecessor.

Check Your Progress

1. Use the appropriate symbol < or > to fill in the following blanks:
   (*i*) (−3 + (−6) ..... (−3) − (−6)
   (*ii*) (−21) − (−10) ..... (−31) + (−11)
   (*iii*) 45 − (−11) ..... (57) + (−4)
   (*iv*) (−25) − (−42) ..... (−42) − (−25)
2. Find the value of:
   (*i*) 12 + (−3) + 5 − (−2).
   (*ii*) 39 − 35 + 7 − (−4) + 21
   (*iii*) −15 − (−2) − 71 − 8 + 6.
3. Evaluate:
   (*i*) $|-13| - |-15|$
   (*ii*) $|35 - 41| - |7 - (-2)|$.
4. Arrange the following integers in ascending order:
   −39, 35, −102, 0, −51, −5, −6, 7.
5. Find the successor and the predecessor of −199.
6. Subtract the sum of −235 and 137 from −152.
7. What must be added to −176 to get −95?
8. What is the difference in height between a point 270 m above sea level and 80 m below sea level?

Activity 2

Objective

To find the sum of two integers.

Pre-requisite knowledge

(*i*) Concept of positive integers, negative integers and zero.
(*ii*) Sum of a positive integer and its negative is zero.

Materials required

(*i*) Coloured chart paper
(*ii*) Grey coloured chart paper
(*iii*) Geometry box
(*iv*) Pair of scissors
(*v*) Fevistick/Gum
(*vi*) A white sheet of paper

Preparation for the activity

1. Cut off 10 squares each of side 1 cm from the coloured chart paper. Let 1 coloured square represent +1.
2. Cut off 10 squares each of side 1 cm from the grey chart paper. Let 1 grey square represent −1.

To perform the activity

We know that sum of a positive integer and its negative is zero *i.e.* one coloured square and one grey square neutralize each other as shown in fig. (*i*).

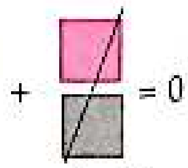


Fig. (*i*)